# Annual Report 2019

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This report covers the research and activities that took place at Centre for Stochastic Geometry and Advanced Bioimaging (CSGB) during 2019 and the remaining funding period January – March 2020. CSGB was established in April 2010 with a grant from the Villum Foundation of DKK 25 million. A second funding period started in 2015, based on an additional grant from the Villum Foundation of DKK 30 million. We are very grateful for the opportunities these grants have given us.

Since the beginning, CSGB has functioned as an inter-institutional collaboration between the Universities of Aarhus, Aalborg and Copenhagen. Four research groups have participated in CSGB: the stochastic geometry group (AU), the section for stereology and microscopy (AU), the spatial statistics group (AAU) and the image section (KU).

A selection of publications, representing research highlights from the second funding period, may be found on page 16-17. This list includes a number of publications, based on fruitful collaboration between the research groups that have participated in CSGB.

A detailed description of the research results obtained January 2019 – March 2020 for each of the six work packages WP1-WP6, that form the research plan of the second funding period of CSGB, is given on page 18-29.

The remaining funding period represented intense international activity for the CSGB staff, involving the organization of (i) Workshop on Diffusion MRI and Stochastic Geometry, (ii) Workshop on Point Processes in Space, Time and Beyond, (iii) 15th International Congress for Stereology and Image Analysis, (iv) 20th Workshop on Stochastic Geometry, Stereology and Image Analysis and (v) Workshop on Manifold and Shape Statistics. More details about the five scientific meetings may be found on page 32-35.

The International Congress for Stereology and Image Analysis took place 27 – 30 May 2019 at Department of Mathematics, Aarhus University, and had about 100 participants from a wide range of disciplines. Five keynote speakers gave longer talks and a number of minisymposia were arranged. A highlight at the conference was the special keynote lecture, given by the grand-old-man of stereology: Luis M. Cruz-Orive.

A PhD competition was organized by the International Society for Stereology and Image Analysis, intended to recognize a PhD thesis in the area of stereology and/or image analysis.

On 27 June 2019, a follow-up meeting between CSGB and the Villum Foundation took place. After an introduction by Director of Science at the Villum Foundation, Thomas Bjørnholt, and myself, CSGB senior staff members presented three projects which are part of work packages WP2, WP3 and WP4. The Foundation also met with the junior staff and, later, with the Centre management. The future of CSGB beyond 2020 was discussed.

The last internal CSGB workshop took place at Brøndums Hotel, Skagen, 7 – 8 November 2019. A total of eighteen workshops of this type have been arranged during 2010-2020. One of the important purposes of these workshops has been to give junior researchers the opportunity to present their research results in an informal forum.

In the following pages, we inform in more detail our colleagues, the Danish funding partner and the Universities of Aarhus, Aalborg and Copenhagen about the organization, research and other centre activities that took place at CSGB in the remaining funding period.

April 2020
Eva B. Vedel Jensen
ORGANIZATION AND STAFF
ORGANIZATION AND STAFF

STAFF

Centre for Stochastic Geometry and Advanced Bioimaging (CSGB) unites four Danish research groups

- **AU-math**: Stochastic Geometry Group, Department of Mathematics, AU
- **AU-bio**: Section for Stereology and Microscopy, Department of Clinical Medicine, AU
- **AAU**: Spatial Statistics Group, Department of Mathematical Sciences, AAU
- **KU**: Image Section, Department of Computer Science, KU

The staff consisted in 2019 of 7 professors, 13 associate professors, 1 assistant professor, 5 postdocs and 16 PhD students, see page 40-41 for details.

RESEARCH PLAN

The research is in the second funding period of CSGB (1 April 2015 – 31 March 2020) organized in six work packages, see the diagram below. The principal investigators of the work packages are also shown in the diagram, just below the title of the work packages.
Aasa Feragen (DTU)
In May 2019, Aasa Feragen was appointed full professor at DTU Compute, Department of Applied Mathematics and Computer Science, Technical University of Denmark. In the second funding period of CSGB (2015-2020), Aasa Feragen has been one of the principal investigators of the Random shapes work package. She has been a CSGB staff member since 2010 where she started as postdoc. In 2019, she obtained support from the Independent Research Fund Denmark within the thematic research programme on digital technologies.

Tom Dela Haije (KU)
During 2019, Tom Dela Haije has obtained funding from the Villum Foundation Experiment Programme. The title of the project is Unraveling the Stochastic Geometry of the Human Brain. He has also been very active in the organization of Workshop on Diffusion MRI and Stochastic Geometry, 20 – 24 January 2019, Sandbjerg, see page 32-33. Tom Dela Haije has been a CSGB postdoc, participating in the work package Random shapes.

Eva B. Vedel Jensen (AU-math)
In 2019, Eva B. Vedel Jensen became honorary member of the International Society for Stereology and Image Analysis. The election as honorary member took place at the 15th International Congress for Stereology and Image Analysis, 27 – 30 May 2019, Aarhus. A report on the congress in general may be found on page 34-35. Eva B. Vedel Jensen has been the director of CSGB since 2010.

Jesper Møller (AAU)
Jesper Møller was appointed visiting professor (an honorary position) at Department of Probability and Mathematical Statistics, Charles University, Prague, in March 2020. Jesper Møller has very close connections to the research group around Viktor Beneš. In the period 21 October – 1 November 2019, he gave five talks at the Department on selected topics in spatial statistics. Jesper Møller has been one of the principal investigators of the work package Spatial and spatio-temporal point processes.

Stefan Sommer (KU)
Stefan Sommer has from May 2019 received additional funding from a project grant in bioscience and basic biomedicine, supported by the Novo Nordisk Foundation. The title of the project is Probabilistic Shape Statistics for Disentangling Subtypes in Alzheimer’s Disease. He was also the prime organizer of the Workshop on Manifold and Shape Stochastics, 23 – 27 February 2020, Sandbjerg. In the second funding period of CSGB (2015-2020), Stefan Sommer has been a principal investigator of the Random shapes work package.
ORGANIZATION AND STAFF

PHD DEFENSES

The following PhD students at CSGB defended their thesis in the period 1 January 2019 – 31 March 2020:

- **March 2019 | Line Kühnel** (KU)  
  Stochastic modelling on manifolds

- **May 2019 | Francisco Cuevas** (AAU)  
  Second order moment properties of point processes and random fields defined on a Euclidean space, a sphere or the product of such spaces

- **October 2019 | Helene Svane** (AU-math)  
  Reconstructing r-regular objects from trinary digital images

- **November 2019 | Heidi Søgaard Christensen** (AAU)  
  Statistics for point processes on linear networks and on the space cross sphere

- **November 2019 | Andreas Dyreborg Christoffersen** (AAU)  
  Iterated and anisotropic marked point processes with a view to the minicolumn hypothesis

- **January 2020 | Anton Mallasto** (KU)  
  Geometric methods in probabilistic modelling

In July 2019, Ninna Vihrs started as PhD student at the AAU group, supported by other sources than CSGB. It is planned that she will participate in a continuation of the minicolumn project.

ORGANIZATION OF INTERNATIONAL WORKSHOPS, CONFERENCES AND PHD COURSES

In the remaining funding period (1 January 2019 – 31 March 2020), the CSGB staff took part in the organization of a number of international workshops, conferences and PhD courses. Some of them are listed on page 32. Below, additional information is given.

- **Markus Kiderlen** (AU-math) obtained substantial additional financial support from Aarhus University Research Foundation for the 20th Workshop on Stochastic Geometry, Stereology and Image Analysis, 2 – 7 June 2019, Sandbjerg. The workshop had about 70 participants from all over the world, see also page 33.

- **Christophe Biscio** (AAU) obtained in 2019 funding from the Carlsberg Foundation to organize the upcoming Conference in Data Science Computing, 4 – 6 August 2020, Aalborg University.

- **Aasa Feragen** (DTU) was program chair at the International Conference on Medical Imaging with Deep Learning, 8 – 10 July 2019, London. For further details, see https://2019.midl.io/. She was also a member of the paper selection committee at the 26th International Conference on Information Processing in Medical Imaging (IPMI), Hong Kong, 2 – 7 June 2019. Furthermore, she was a member of the program committee of the Medical Imaging meets NeurIPS Workshop, 14 December 2019, Vancouver. See also https://sites.google.com/view/med-neurips-2019.
FOLLOW-UP MEETING - 27 JUNE 2019

On 27 June 2019, a follow-up meeting took place at Department of Mathematics, Aarhus University, between CSGB and the Villum Foundation.

From the Villum Foundation, the following participated: Director of Science Thomas Bjørnholm, Professor Anja Boisen (DTU, member of the Foundations’ board), Professor Christian S. Jensen (AAU, member of the Foundation’s board) and Senior Adviser Michel M.H. Kristensen. From CSGB, 26 staff members participated, representing about 65% of the staff. In addition, head of Department of Mathematics, AU, Jacob Schach Møller participated in the meeting.

After an introduction by Director of Science Thomas Bjørnholm, Eva B. Vedel Jensen gave a brief overview of CSGB, including the present staff, centre activities, status of the work packages and selected highlights since the last follow-up meeting. This overview was followed by the presentation of three projects at CSGB:

• **Markus Kiderlen** (AU-math): *On the precision of stereological volume estimators from planar sections*
• **Rasmus Waagepetersen** (AAU): *Interacting curves in complex environments*
• **Stefan Sommer** (KU): *Geometric statistics: when space curves and estimation becomes smeary*

Afterwards, the Foundation met with the junior staff and, later, with the Centre management. The future of CSGB beyond the present funding period (2015-2020) was discussed.

SELECTED HIGHLIGHTS

• During 2019, a number of papers written by members of the **AAU group** has been published or accepted for publication in absolute top international statistical journals, including *Annals of Statistics* and *Journal of the American Statistical Association*.
• **Rune Kok Nielsen** (KU) gave an oral presentation at the *International Conference on Medical Image Computing and Computer-Assisted Intervention (MICCAI) 2019*. This conference is the most prestigious conference in medical image analysis. The success rate for oral presentations was 3.3%.
• **Tom Dela Haije** (KU) won a best paper award at *CDMRI 2019*.

NEW COLLABORATIONS

**Aasa Feragen** (DTU) has established collaboration with Simone Vantini and Anna Calissano, MOX Laboratory for Modeling and Scientific Computing, Politecnico Milano.

**Stefan Sommer** (KU) started collaboration with Stephan Huckemann and Benjamin Eltzner, Institute for Mathematical Stochastics, Georg-August-Universität, Göttingen, concerning central limit theorems for diffusion means on manifolds.

**Jakob G. Rasmussen** (AAU) has established collaboration with Leif Sörnmo and Alba Martin-Yebra, Department of Biomedical Engineering, Lund University.

**Christophe Biscio** (AAU) started in 2019 a collaboration with researchers from INRAE, Avignon, on asymptotic properties of standard estimators in spatial statistics.

SOFTWARE

**Rune Kok Nielsen** (KU) and coworkers have developed the software package TopAware (https://github.com/RuneKokNielsen/TopAware/). State-of-the-art algorithms for deformable registration restrict to diffeomorphisms to regularize an otherwise ill-posed problem. A novel, piecewise-diffeomorphic deformation framework is presented which models topological changes as explicitly encoded discontinuities in the deformation fields. The entire model is GPU-implemented. See also page 20-21.
RESEARCH

In the second funding period of CSGB, the research has been organized in six work packages

WP1: Valuation theory
WP2: Random shapes
WP3: Spatial and spatio-temporal point processes
WP4: Point processes in bioimaging
WP5: Statistics for stochastic geometry models
WP6: Algorithms

A detailed description of the research results obtained in the remaining funding period 1 January 2019 - 31 March 2020 may be found on page 18-29.

Below, we present publication highlights for the full second funding period. The diagramme indicates some of the publications, involving collaboration between the participating research groups.

PUBLICATION HIGHLIGHTS - Second funding period (2015-2020)


Researchers
Johnny B. Andersen
Karl-Anton Dorph-Petersen
Rikke Krog Eriksen
Eva B. Vedel Jensen
Markus Kiderlen
Nick Yin Larsen
Jens R. Nyengaard
Andrew du Plessis
Mads Stehr
Anne Marie Svane
Johanna F. Ziegel

Valuation theory

During the funding period of CSGB (2010-2020), an important focus point has been tensor valuations and rotational integral geometry. Motivated by applications in microscopy, rotational Crofton formulae and principal rotational formulae for Minkowski tensors have been developed during this period. Many of these results are reviewed in Jensen & Kiderlen (2017a). See also the overview Jensen & Kiderlen (2017b, eds.) of the modern theory of tensor valuations, presented in the Springer Lecture Notes in Mathematics Series. A book project (Jensen, 2020+) has been initiated in 2019 with the purpose to make these research results available to the stochastic geometry community in a broad sense. Applications in microscopy will be included in the planned monograph.

In 2019, CSGB staff members Rikke Krog Eriksen and Markus Kiderlen finalized a comprehensive paper on uniqueness of measurement functions (Eriksen & Kiderlen, 2020a). This research was originally motivated by the observation that two apparently very different measurement functions were suggested in local stereology for estimation of surface area. In Eriksen & Kiderlen (2020a), the question of uniqueness of measurement functions in Crofton’s formula is investigated. Crofton’s formula states for compact convex subsets $K$ in $\mathbb{R}^n$ that

$$\int_{A(n,k)} V_j(K \cap E) \mu_k(dE) = \alpha_{n,j,k} V_{n+j-k}(K),$$

where $A(n,k)$ is the space of $k$-dimensional affine subspaces of $\mathbb{R}^n$, $V_j$ is the $j$th intrinsic volume, $\mu_k$ is the motion invariant measure on $A(n,k)$ and $\alpha_{n,j,k}$ is a known constant. The question is now whether there exist other functionals $\varphi$ (possibly under additional restrictions on $\varphi$) for which

$$\int_{A(n,k)} \varphi(K \cap E) \mu_k(dE) \propto V_{n+j-k}(K).$$

It turns out that this easily posed question has a rather involved answer.

In Eriksen & Kiderlen (2020a), the focus is on the subclass of local functionals $\varphi$. The local property is formally defined in Weil (2017). An extension of Weil (2017, Theorem 2.1) to local functionals on compact convex sets of dimension at most $k \leq n - 1$ is shown in Eriksen & Kiderlen (2020a) and a decomposition of such functionals into homogeneous local functionals is given. The conclusion is that the measurement function is not unique, but within the class of local functionals a characterization of the solutions to (1) can be given for $k = 1$. Likewise, for $k = 2$, a characterization is available for $\varphi$ amongst even local functionals. Additional functionals are constructed for $k > 2$.

The recent research on stereology of tensors has been published in Larsen et al. (2019). In this paper, a simple local stereological method of estimating particle shape from vertical sections is presented to scientists working in microscopy. The method uses measurements in a vertical optical plane, passing through a reference point of each sampled particle. The measurements performed on a sampled particle are illustrated in Figure 1. A systematic set of alternating half lines (indexed by $i$) is used on each sampled particle profile, generated by the vertical plane. If we let $l_{ij}$ be the distance from the $j$th intersection point on the $i$th half line to the vertical axis, see Figure 1, it is needed to determine the so-called squared ray distance for the $i$th half line

$$l_i^2 = \sum_{j \text{ even}} l_{ij}^2 - \sum_{j \text{ odd}} l_{ij}^2.$$  

Likewise, the power-4 ray distance is determined

$$l_i^4 = \sum_{j \text{ even}} l_{ij}^4 - \sum_{j \text{ odd}} l_{ij}^4.$$  

The estimator of particle shape is based on estimators of volume tensors of the sampled particle which are simple functions of the following quantities

$$\sum_i l_i^2, \sum_i l_i^4, \sum_i l_i^6, \sum_i z_i^2 l_i^2.$$  

Here, $z_i$ is the (signed) distance from the reference point (denoted $O$ in Figure 1) to the $i$th half line.

The method is superior to earlier, more time-consuming methods, based on observation in several optical planes. In
a model-based setting, the new method requires that the particle distribution is invariant under rotations around the vertical axis. For a detailed discussion of this assumption, see Hasselholt et al. (2019). As a new contribution, a dual design-based approach is developed in the paper Larsen et al. (2019). In the design-based case, it is not needed to assume restricted isotropy, and the method provides information about an index of elongation of the particles in the direction of the vertical axis. We have also managed to generalize the method to $\gamma$-dimensional space, see the forthcoming publication Eriksen & Kiderlen (2020b). A review, with emphasize on the importance of vertical sections in stereology, may be found in Jensen (2020).

The research Bianchi et al. (2019) on mappings that can be characterized as rearrangements, in particular polarizations, was also finalized in 2019.

References


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<th>$l_{i2}$</th>
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Figure 1

Illustration of the measurements on a sampled particle profile. A systematic set of alternating half lines, perpendicular to the vertical axis (VA), is used. The intersection points on a given half line are numbered according to decreasing distance to VA, using number 0 for the most distant intersection point. We let $l_{ij}$ be the distance from the $j$th intersection point on the $i$th half line to VA and $z_i$ is the (signed) distance from the reference point (denoted $O$) to the $i$th half line. The measurements in the table are in arbitrary units and refer to the shown particle profile. The distance between neighbour half lines is $t = 1.2$. For details, see Larsen et al. (2019).
Random shapes

For data taking values in non-linear spaces, a number of advances were made in the remaining funding period. Wrapped Gaussian Processes (WGP) were utilized to define probabilistic submanifold learning algorithms (Mallasto et al., 2019). More precisely, when data is known to reside on a manifold, for instance due to invariances or constraints, a WGP latent variable model is used in Mallasto et al. (2019) to learn a stochastic data manifold, which is ensured to satisfy the constraints or invariances by being constrained to lie within the manifold. Beyond the realm of manifolds, we started collaborating with Anna Calissano and Simone Vantini at the MOX Laboratory at Politecnico di Milano on non-linear statistics for network data in mobility planning.

Geometry also enters into statistical modelling through geometric properties of the models themselves, a fact that has been used in several subprojects. State-of-the-art image registration assumes, largely for the sake of regularization, that all anatomies are diffeomorphic deformations of a standard anatomy. However, anatomy varies even in healthy subjects. In diseased subjects, matter appears in the form of tumors, bleeds and cavities. It disappears e.g. via surgery or atrophy. We have made the first steps towards solving these problems by introducing piecewise diffeomorphic models that allow for the introduction or removal of matter (Kok Nielsen et al., 2019). This paper was awarded an oral presentation at the highest ranking conference in medical imaging, MICCAI, with an acceptance rate of 3.3%.

In Dela Haije et al. (2020), constrained optimization is utilized to ensure that local microstructural models estimated from diffusion magnetic resonance images (dMRI) actually satisfy known geometric constraints. This is hugely important. State-of-the-art microstructural models are shown to defy natural laws in large portions of the brain, potentially affecting down-stream analysis of both microstructure and connectivity.

Also related to local microstructural models in dMRI, Dela Haije & Feragen (2019) use rotational invariants to obtain unbiased estimates of the so-called response functions in constrained spherical deconvolution. This paper also won the best paper award.

In Jensen et al. (2019), an information-theoretic approach to registration of diffusion weighted images with explicit optimization over the orientational scale is presented. The LOR-DWI density-based hierarchical scale-space model is extended to non-rigid deformations, and it is shown that the formulation provides intrinsic regularization through the orientational information.

When the data are elements of non-linear spaces, such as differentiable manifolds, methodology for simulating bridge processes is lacking. In particular, in cases where the transition probability densities are intractable, it is of interest to use simulation schemes that can numerically approximate the true densities. In Haighaard Jensen et al. (2019), a method for simulating diffusion bridges on the flat torus is proposed. This specific case serves as an example of the more general setting of simulating diffusion bridge processes on Riemannian manifolds.

Frank van der Meulen from TU Delft spent a six month sabbatical in Copenhagen working on bridge sampling on non-linear spaces. The work has resulted in successful transfer of sampling techniques from the Euclidean setting to examples of manifold valued data. In particular, the developed scheme covers stochastic models for landmark manifolds for which no prior sampling scheme has been available.

Data is often observed without ordering implying that a sample of multiple observations are naturally modelled in configuration spaces, the n-fold product modulo permutations. When the data space itself is a manifold, this quotient space has interesting geometric structure. Together with Philipp Harms, Xavier Pennec and Peter Michor, we have explored geometric statistics on configuration spaces, including proving a central limit theorem for unordered samples without the common assumption of independence between the samples.

Many aspects of the work on geometric statistics, performed by the KU group associated with CSGB, has been described in the book Pennec et al. (2020) on Riemannian Geometric Statistics in Medical Image Analysis, edited by Stefan Sommer together with Xavier Pennec and Tom Fletcher. The book contains multiple contributions from the KU group.
During 2019, the research on a geometric framework for stochastic shape analysis (Arnaudon et al., 2019), deterministic group tractography (Holm et al., 2019), deep learning numerics (Kühnel et al., 2019) and principal component analysis on manifolds (Sommer, 2019) has been published.

References


Figure 1
Simulated DWI samples. Left: The raw DWI signal. Right: The reconstructed diffusion ODFs that follow anisotropic diffusion. The red lines indicate fiber orientations. For details, see Jensen et al. (2019).
Researchers
Christophe A.N. Biscio
Achmad Choiruddin
Heidi Seggaard Christensen
Andreas Dyreborg Christoffersen
Kristian Bjørn Hessellund
Christian Hirsch
Jesper Møller
Francisco Andrés Cuevas Pacheco
Jakob Gulddahl Rasmussen
Ege Holger Rubak
Anne Marie Svane
Rasmus Plenge Waagepetersen

Spatial and spatio-temporal point processes

A highlight was the acceptance in the very top journal Annals of Statistics of the 35 pages long paper by CSGB researchers Jesper Møller and Jakob Gulddahl Rasmussen (in collaboration with Ethan Anderes) on isotropic covariance functions on linear networks and their extension to graphs with Euclidean edges (Anderes et al., 2019). The covariance functions are isotropic in the sense that they only depend on a new metric that extends the classical resistance metric on the vertices of a weighted graph to both the vertices and edges of any graph with Euclidean edges.

The research on the use of persistence diagrams for analyzing point patterns resulted in two publications. The diagram captures geometric structures such as point clusters and voids. For this reason, the accumulated persistence function has been suggested in Biscio & Møller (2019) as a functional summary statistics for point patterns. While the geometric robustness of the persistence diagram is well-studied, only limited results on its statistical properties are available. In a recent paper (Biscio et al., 2019), CSGB researchers Christophe Biscio and Anne Marie Svane, together with Christian Hirsch, have studied the use of topological data analysis for assessing the goodness of fit of point process models. In particular, they derive a functional central limit theorem for the persistence diagram of a planar point pattern, which allows testing the goodness of fit of a given point process model, using the accumulated persistence function. In a future research project, they seek to extend the results to marked point processes.

Rasmus Waagepetersen and collaborators finished and published in 2019 a paper on second-order variational estimating equations for spatial point processes (Coverjolly et al., 2019). A focus point is here pair correlation function estimation. In the case of log linear parametric models for pair correlation functions, it is demonstrated that the variational equations can be applied to construct estimating equations with closed form solutions for the parameter estimates. See also Jalilian et al. (2019).

A new method for analysis of multivariate point process data is suggested in Hessellund et al. (2019). Semi-parametric models for the intensity functions are proposed. A multinomial conditional composite likelihood function is introduced for estimation of intensity function regression parameters and the asymptotic joint distribution of the resulting estimators is derived under mild conditions. The asymptotic covariance matrix depends on the cross pair correlation functions of the multivariate point process. Standardized residual plots and predictive probability plots are constructed for the validation of the model.

Rasmus Waagepetersen and coworkers have also published papers on regularized inference for highly multivariate point processes (Choiruddin et al., 2020), inference for stochastic radio channel models (Hirsch et al., 2020) and adaptive estimating functions for determinantal point processes (Lavancier et al., 2019).

The research on leverage and influence diagnostics (Baddeley et al., 2019), ϵ-mixing point processes (Biscio & Waagepetersen, 2019), Voronoi intensity estimators (Mehdi Moradi et al., 2019), K-functions for space curves (Sporring et al., 2019) and stochastic quasi-likelihood (Xu et al., 2019) has been published or accepted for publication. This research has been described in CSGB Annual Report 2018. Dvořák et al. (2019) contains a quick inference method for log Gaussian Cox processes.

References


Figure 1
Alpha-complex with alive (blue) and dead (red) loops marked (left) and associated persistence diagram (right). For details, see Biscio et al. (2019).
Point processes in bioimaging

The project on improved variance prediction in stereology has been finalized by CSGB PhD student Mads Stehr and Markus Kiderlen (Stehr & Kiderlen, 2019). This research was originally motivated by the observation that the variance of Cavalieri type stereological estimators may be severely inflated if the sampling points are unequally spaced, see Ziegel et al. (2010) and references therein. The results obtained in Stehr & Kiderlen (2019) apply to numerical integration in general when sampling nodes are random. It is suggested to use Newton-Cotes quadrature to exploit the smoothness properties of the integrand. Under some integrability conditions on the typical point-distance, it is shown that Newton-Cotes quadratures based on a stationary point process on $\mathbb{R}$ yield unbiased estimators for the integral and that the aforementioned variance inflation can be avoided if a Newton-Cotes quadrature of sufficiently high order is applied.

CSGB PhD student Heidi Sagaard Christensen together with Jesper Møller completed in 2019 their work on modelling spine locations on dendrite trees, using inhomogeneous Cox point processes (Christensen & Møller, 2019). Dendritic spines are small protrusions on the dendrites of a neuron. In neuroscience, the spatial distribution of the spines on dendrite trees is of interest, as changes in this distribution may be linked to changes in cognitive processes. The spine locations can be viewed as a point pattern on the dendrite tree and thus analyzed, using point process theory for linear networks.

There are only a limited number of point process models available for linear networks. An exception is directed acyclic linear networks, for which both regular and clustered models can be defined by a generalization of the conditional intensity function for temporal point processes (Rasmussen & Christensen, 2020). In Christensen & Møller (2019), a new class of Cox process models on a linear network is suggested. Moreover, the use of minimum contrast and composite likelihood procedures is demonstrated, and new empirical summary functions are introduced. The distribution of spine locations on six different dendrite trees from mouse neurons is analyzed, using the new methodology.

The minicolumn hypothesis in neuroscience claims that neurons and other brain cells have a columnar arrangement, perpendicular to the surface of the brain. This hypothesis has been extensively studied in the biological literature, see e.g. Buxhoeveden & Casanova (2002), Rafati et al. (2016) and references therein. In 2019, Heidi Sagaard Christensen together with Jesper Møller and CSGB PhD student Andreas Dyreborg Christoffersen submitted a paper on modelling columnarity of pyramidal cells in the human cerebral cortex (Christoffersen et al., 2019). In the paper, a new hierarchical point process model is suggested, consisting first of a generalized shot noise Cox process in the xy-plane, providing cylindrical clusters, and next of a Markov random field model on the z-axis. Several cases of these hierarchical point processes are fitted to two pyramidal cell datasets. The location of a pyramidal cell is represented by its nucleolus. The data also includes the local orientation of the cell, determined by the direction from the nucleolus to the apical dendrite’s position. However, since it was found in Møller et al. (2019) that locations and orientations of the cells were independent, the focus of Christoffersen et al. (2019) is on modelling the locations. The new model provides a better fit to these datasets than the so-called Poisson line cluster point process model, earlier suggested in Møller et al. (2016).

New spatio-temporal point process models for super resolution fluorescence microscopy data have been developed in the remaining funding period. The models account for the blinking characteristics of fluorescent proteins, and analysis tools have been developed that allow for estimation of both model and nuisance parameters, including the fraction of noise points and number of re-appearances per fluorescent protein.

In 2019, the research involving photoactivated localization microscopy was published in Arnspang et al. (2019). In continuation of this research, an analysis of renal aquaporin-2 (AQP2) in collecting duct principal cells has been carried out as part of a collaborative research project. The data consists of noisy maps of protein locations, observed in 3D.
As a helpful assistant, I would be happy to help you with your request. Please provide me with the specific document or section you would like me to convert into a plain text representation.
Statistics for stochastic geometry models

The general aim of this work package has been to develop new statistical inference procedures for stochastic geometry models. The focus has been on Monte Carlo envelope tests for spatial models, on asymptotics for excursion sets and on estimation of sample spacing.

During the second funding period of CSGB (2015-2020), three papers on envelope testing have been published by CSGB researcher Ute Hahn and collaborators, see Mrkvička et al. (2016, 2017) and Myllymäki et al. (2017). The paper Myllymäki et al. (2017) in the prestigious Journal of the Royal Statistical Society B has already received 112 citations, according to Google. The envelope method can be extended for comparison of groups of functional data.

Asymptotics of excursion sets has been studied for Lévy-based random fields. In 2019, the CSGB group (Stehr & Rønn-Nielsen, 2019) has completed a study of Lévy-based models depending on both space and time, generalizing the models appearing in Rønn-Nielsen & Jensen (2016, 2017). The space-time random field

\[ X = (X_{t,v})_{(v,t) \in \mathbb{R}^d \times [0, T]} \]

is the Lévy-driven moving average model defined by

\[ X_{t,v} = \int_{\mathbb{R}^d \times (-\infty, v]} f(|v - u|, t - s) M(du, ds), \]

where \( M \) is an infinitely divisible, independently scattered random measure on \( \mathbb{R}^{d+1} \), \( f \) is some kernel function, and \( \mathbb{B} \) and \( (v, T) \) are compact index sets. The index \( v \) refers to the spatial position, while \( t \) is interpreted as time. The random field is a causal model since \( X_{t,v} \) only depends on the restriction of \( M \) to \( \mathbb{R}^d \times (-\infty, t] \).

Such Lévy-driven moving average models provide a flexible and tractable modelling framework. Examples of applications, that include both time and space, are modelling of turbulent flows (Barndorff-Nielsen & Schmiegel, 2004) and growth processes (Jónsdóttir et al., 2008). Purely spatial models have been used for modelling of Cox point processes (Hellmund et al., 2008) and brain imaging data (Jónsdóttir et al., 2013; Rønn-Nielsen et al., 2017). The latter paper deals with estimation of sample spacing. In Rønn-Nielsen & Jensen (2019), central limit theorems are proved for mean and variogram estimators in Lévy-driven moving average models.

In Stehr & Rønn-Nielsen (2019), it is assumed that the Lévy measure \( \rho \) of the random measure \( M \) has a convolution equivalent right tail. Under regularity conditions, it is shown that certain functionals of the field \( X \) have a right tail that is equivalent to the tail of the underlying Lévy measure, i.e. there exist known constants \( C \) and \( \epsilon \) such that

\[ P(\Psi(X) > x) \sim C \rho(x/\epsilon, \infty), \]

as \( x \to \infty \). The simplest example of a functional \( \Psi \) considered is \( \Psi(X) = \sup_{v \in \mathbb{B}} X_{v,t} \). Another example relates to the spatial excursion set at level \( x \) and time \( t \)

\[ A_{x,t} = \{ v \in \mathbb{B} : X_{v,t} > x \}. \]

It is shown under further regularity conditions that the asymptotic probability that there exists a time point \( t \) for which the excursion set at level \( x \) contains a ball of fixed radius has a tail that is equivalent with the tail of \( \rho \). In a third example, a similar result is shown for the probability that an average of the field over a time interval and a set of spatial positions exceeds the level \( x \).

The results obtained in Stehr & Rønn-Nielsen (2019) are generalizations of those obtained in the earlier papers Rønn-Nielsen & Jensen (2016, 2017) for spatial random fields. In Fasen (2008), results for a moving average process on \( \mathbb{R} \), obtained as an integral with respect to a Lévy process with convolution equivalent tail, are derived. Here, the process \( \{X_t\}_{t \in [0, T]} \) is given by

\[ X_t = \int_{-\infty}^{t} f(t - s) M(ds), \]

where \( M \) is a convolution equivalent Lévy measure on \( \mathbb{R} \). In agreement with the results in Stehr & Rønn-Nielsen (2019), it is derived in Fasen (2008) that \( \sup_{t} X_t \) has a tail that is asymptotically equivalent to the tail of the underlying Lévy measure.
References


Figure 1
Two excursion sets (middle and lower) for the spatial random field shown in the upper plot. In Stehr & Rønn-Nielsen (2019), excursion sets for space-time random fields are studied.
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Algorithms

In the remaining funding period of CSGB, the focus has been on reconstruction and segmentation of objects from digital grey-value images.

In the PhD thesis Svane (2019), reconstruction of both 2D and 3D objects from their digital images are considered. When working with reconstruction from digital images, it is necessary to put some constraints on the objects, being considered, because the digitization process removes features that are small compared to the pixel (voxel) size. In Svane (2019), the objects are assumed to be \( r \)-regular. Intuitively, the notion of \( r \)-regularity ensures that an \( r \)-regular object cannot have a boundary with too many curlicues, or have sections with too narrow necks.

A number of authors have formulated such regularity conditions in slightly different ways (Federer, 1959; Serra, 1983; Stelldinger & Köthe, 2005; Duarte & Torres, 2015).

The digital images are constructed by placing the \( r \)-regular object on a grid and then colouring the grid squares (or cubes, when we work with 3D images) black if they are completely inside the object, white if they are completely outside the object, and grey otherwise. In this way, a digital image is obtained with just three colours: black, white and grey. Such an image is called a trinary image.

The paper Svane & du Plessis (2019), which is part of the PhD thesis Svane (2019) and also available at arXiv, deals with reconstruction of 2D \( r \)-regular objects from their trinary images. The main result of this paper is the following. If we let \( d \) be the pixel side length, it is possible for an \( r \)-regular object \( X \) in 2D with \( d\sqrt{2} < r \) to construct an object such that the boundaries of the original object and the reconstructed object are closer than \( d \) apart in Hausdorff distance. In Figure 1 and 2, the reconstruction of a 2D object from a trinary image is illustrated. Whenever two grey pixels share an edge, an auxiliary point is placed on that edge, see Figure 1, left. Then, a unique circle arc is determined through each three consecutive auxiliary points, see Figure 1, right. Each set of two consecutive auxiliary points is thereby connected by two circle arcs. Interpolating between these two curves for every set of two consecutive auxiliary points, a continuous curve is constructed, forming the boundary of the reconstructed object, see Figure 2.

In the paper Svane & Feragen (2019), which is also part of the PhD thesis Svane (2019), reconstruction of objects from their noisy images at low resolution is considered. A method is proposed for removing noise by comparing the observed \( 3 \times 3 \) pixel configurations in the noisy image to the \( 3 \times 3 \) trinary image configurations that may occur in an ideal digital image of an \( r \)-regular object. The solution provided is a globally optimal algorithm that is computationally very demanding. For this reason, a local approach for reconstructing the trinary image is also proposed in Svane & Feragen (2019).

A 2D and 3D variational segmentation approach based on similarity invariant, i.e. translation, scaling and rotation invariant, shape regularisers is presented in Hansen & Lauze (2019). Shape moments of order up to 2 for shapes with limited symmetries can be combined to provide a shape normalization for the group of similarities. In order to obtain a segmentation objective function, a two-means or two-local-means is added to it. Segmentation is then obtained by standard gradient descent. The capabilities of the approach is demonstrated on a series of experiments.
Figure 1
A reconstruction of a trinary image is obtained by first plotting auxiliary points on the boundaries of the grey pixels (left) and then connecting three consecutive auxiliary points by circle arcs (right).

Figure 2
Smooth curves (yellow) are obtained by interpolation of the circular arcs, shown in Figure 1, right. The black curves are the boundaries of the original object.

References


CENTRE ACTIVITIES
OVERVIEW
PAST AND PLANNED INTERNATIONAL ACTIVITIES

INTERNATIONAL CONFERENCES, SYMPOSIA AND WORKSHOPS

• Workshop on Diffusion MRI and Stochastic Geometry
  20 – 24 January 2019, Sandbjerg

• Workshop on Point Processes in Space, Time and Beyond
  13 – 16 May 2019, Skagen

• 15th International Congress for Stereology and Image Analysis
  27 – 30 May 2019, Aarhus

• 20th Workshop on Stochastic Geometry, Stereology and Image Analysis
  2 – 7 June 2019, Sandbjerg

• Workshop on Quantitative Microscopy
  12 – 16 August 2019, Bern

• Workshop on Manifold and Shape Stochastics
  23 – 27 February 2020, Sandbjerg

INTERNATIONAL PHD AND MASTER COURSES

• Quantitative Medical Graphics
  27 February 2019, Aarhus

• Interdisciplinary Summer School on Neuroimaging
  16 – 26 July 2019, Aarhus

• PhD Summer School on Generative Models
  12 – 16 August 2019, Blokhus

• PhD Course on Design and Analysis of Experiments
  10 – 24 October 2019, Aalborg

• PhD Course on Selected Topics in Spatial Statistics
  21 October – 1 November 2019, Prague

• PhD Course in Neuroscience
  6 – 10 November 2019, Beijing

• PhD Course on Bayesian Statistics, Simulation and Software
  4 – 11 December 2019, Aalborg

Photo Ute Hahn.
Scope of the workshop
The focus of this workshop was to strengthen the connections between diffusion MRI and stochastic geometry. Stochastic geometry is widely used in a variety of fields, including astronomy, geology and telecommunications, but its application in diffusion MRI has so far remained limited. There is a great potential in enhancing the use of stochastic geometry tools in diffusion MRI, where different types of challenging random geometrical patterns are observed. At the workshop, leading researchers from the two communities were brought together to discuss recent advances and connections between the fields.

Structure of the workshop
The workshop had longer keynote talks and shorter invited talks, as well as a poster session.

Organizers
The workshop was organized by CSGB staff members:
- Tom Dela Haije (KU)
- Aasa Feragen (DTU)
- Eva B. Vedel Jensen (AU-math)
- Oddbjørg Wethelund (AU-math)

Organizers and financial support
The organizers of this workshop were CSGB staff members from AU-math:
- Ute Hahn
- Eva B. Vedel Jensen
- Markus Kiderlen
- Oddbjørg Wethelund

Financial support was obtained from the Aarhus University Research Foundation.

Scope of the workshop
This workshop was the 20th in a biennial series of meetings, bringing together researchers and practitioners working with random geometric objects. The goal of the workshop was to promote advances in all branches of stochastic geometry and related fields including, but not limited to, the theory of point processes, integral geometry, stereology, discrete and continuum percolation and random graphs, spatial statistics, bioimaging and all other application areas of spatial stochastic modelling.

Structure of the workshop
The workshop had longer talks by invited speakers and shorter contributed talks by the participants, as well as a poster session.
The 15th International Congress for Stereology and Image Analysis took place 27 – 30 May 2019 at Department of Mathematics, Aarhus University, Denmark (http://conferences.au.dk/icsia2019/).

On Monday morning, the organizing staff welcomed about 100 researchers from a wide range of disciplines and Jens R. Nyengaard opened the conference.

15TH INTERNATIONAL CONGRESS FOR STEREOLOGY AND IMAGE ANALYSIS
27 – 30 MAY 2019, AARHUS UNIVERSITY, DENMARK

In addition, a number of minisymposia were arranged (organizer(s) in parenthesis):

• Advanced quantification of lung tissue (Matthias Ochs and Christian Mühlfeld)
• Digital pathology (Torben Steiniche)
• Fibre processes and orientation maps (Jon Sporring)
• Image analysis and stochastic modelling of materials (Claudia Redenbach)
• Machine learning techniques for image segmentation and object detection (Volker Schmidt)
• Mathematical morphology in the deep learning era (Jesus Angulo and Santiago Velasco-Forero)
• Neurostereology and image analysis (Dorothy Oorschot and Jens R. Nyengaard)
• Sampling in stereology (Marcos Cruz)
• Stochastic modelling of 3D microstructures for virtual materials testing (Volker Schmidt)

SCIENTIFIC PROGRAMME

The programme of the conference was organized in two parallel sessions.

Five keynote speakers gave longer talks:

Michael Klatt
(Princeton)

Mari Myllymäki
(Helsinki)

Emilio Porcu
(Newcastle)

Katja Schladitz
(Kaiserslautern)

Fei Sun
(Beijing)

Photos by CSGB staff.
The International Society for Stereology and Image Analysis organized a PhD competition intended to recognize a PhD thesis conducted in the area of stereology and/or image analysis. Johannes Österreicher won this competition and gave a lecture about his PhD thesis in plenum.

A highlight at the conference was the special keynote lecture, given by the grand-old-man of stereology: Luis M. Cruz-Orive. The title of the lecture was Stereology – personal recollections.

SOCIAL EVENTS

The conference was organized by Ute Hahn, Eva B. Vedel Jensen, Markus Kiderlen, Jens R. Nyengaard and Oddbjørg Wethelund.

A conference dinner was arranged at the art museum in Aarhus, ARoS. Prior to the dinner, it was possible to participate in a guided tour at ARoS. Leading members of the International Society for Stereology and Image Analysis were photographed at ARoS.

The welcome reception on Monday 27 May took place at the Greenhouses in the Botanical Garden, Aarhus. A second group photo was taken here.

A conference dinner was arranged at the art museum in Aarhus, ARoS. Prior to the dinner, it was possible to participate in a guided tour at ARoS. Leading members of the International Society for Stereology and Image Analysis were photographed at ARoS.

Organization

The conference was organized by Ute Hahn, Eva B. Vedel Jensen, Markus Kiderlen, Jens R. Nyengaard and Oddbjørg Wethelund.


EIGHTEENTH INTERNAL CSGB WORKSHOP
Brøndums Hotel, Skagen, 7 – 8 November 2019

The last internal CSGB workshop took place at Brøndums Hotel, Skagen, 7 – 8 November 2019. During the full funding period 2010-2020 of CSGB, these workshops have played an important role. The present status of the CSGB research projects has been discussed at these workshops and plans for further progress have been made. Furthermore, the junior researchers have had the opportunity to present their research results in an informal forum.


SELECTED TALKS BY CSGB STAFF
1 January 2019 – 31 March 2020

Workshop on Diffusion MRI and Stochastic Geometry
Sandbjerg | 20 – 24 January 2019

Ute Hahn: Simulation based testing for stochastic geometry models

Workshop on Diffusion MRI and Stochastic Geometry
Sandbjerg | 20 – 24 January 2019

Markus Kiderlen: Fibre processes in stochastic geometry

15th International Congress for Stereology and Image Analysis
Aarhus | 27 – 30 May 2019

Tom Dela Haije: Imaging brain fibers with diffusion MRI

15th International Congress for Stereology and Image Analysis
Aarhus | 27 – 30 May 2019

Mads Stehr: Asymptotic variance of Newton-Cotes quadratures based on randomized sampling points

20th Workshop on Stochastic Geometry, Stereology and Image Analysis
Sandbjerg | 2 – 7 June 2019

Jesper Møller: The structure of stationary time series and point processes when constructing singular distribution functions

20th Workshop on Stochastic Geometry, Stereology and Image Analysis
Sandbjerg | 2 – 7 June 2019

Anne Marie Svane: Testing goodness of fit for point processes via topological data analysis

Stochastic Geometry Days
Avignon | 17 – 21 June 2019

Rasmus Waagepetersen: Multinomial logistic regression and regularized estimation for multivariate point processes

XI International Conference of Mathematical Physics in Armenia
Yerevan | 2 – 7 September 2019

Jesper Møller: Determinantal point processes and their usefulness in spatial statistics

Talk at Sino-Danish Center
Beijing | 3 September 2019

Jens R. Nyengaard: Glia cells

DALI 2019 – DATA, Learning and Inference
San Sebastian | 3 – 6 September 2019

Aasa Feragen: Uncertainties in diffusion MRI

DALI 2019 – DATA, Learning and Inference
San Sebastian | 3 – 6 September 2019

Aasa Feragen: Uncertainty quantification for manifold valued models

7th Annual Neuroscience & Neuroimaging Symposium
Beijing | 4 November 2019

Jens R. Nyengaard: Omics in neuroscience

CMStatistics 2019
London | 14 – 16 December 2019

Christophe Biscio: A general central limit theorem and a subsampling variance estimator for \( \chi \)-mixing point processes

CMStatistics 2019
London | 14 – 16 December 2019

Aasa Feragen: The geometry of graph space: Towards graph-valued statistics

CMStatistics 2019
London | 14 – 16 December 2019

Louis Gammelgaard Jensen: Doubly stochastic point processes in time and space as a model for photoactivated localization microscopy data

Quantitative Bioimaging Conference
Oxford | 6 – 9 January 2020

Rasmus Waagepetersen: Inferring correlations in inhomogeneous and multitype point patterns

Workshop on Manifold and Shape Stochastics
Sandbjerg | 23 – 27 February 2020

Jesper Møller: Statistics for point processes on the \( d \)-dimensional unit sphere

Workshop on Manifold and Shape Stochastics
Sandbjerg | 23 – 27 February 2020

Mathias Højgaard Jensen: Simulation of conditioned diffusions on Riemannian manifolds
APPENDIX

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Photo: Petra Steiner, Daniela Mayer, CSGB staff

Design and layout: DesignZone, Daniela Mayer, www.johan-k.de

Printed in Denmark by SUN-Tryk

ISSN: 1904-9404

www.csgb.dk