CENTRE FOR **STOCHASTIC GEOMETRY**
AND ADVANCED **BIOIMAGING**

Annual Report 2019

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Annual Report 2019, published May 2020

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INTRODUCTION

This report covers the research and activities that took place at Centre for Stochastic Geometry and Advanced Bioimaging (CSGB) during 2019 and the remaining funding period January – March 2020. CSGB was established in April 2010 with a grant from the **Villum Foundation** of DKK 25 million. A second funding period started in 2015, based on an additional grant from the Villum Foundation of DKK 30 million. We are very grateful for the opportunities these grants have given us.

Since the beginning, CSGB has functioned as an **inter-institutional collaboration** between the Universities of Aarhus, Aalborg and Copenhagen. Four research groups have participated in CSGB: the stochastic geometry group (AU), the section for stereology and microscopy (AU), the spatial statistics group (AAU) and the image section (KU).

A selection of publications, representing **research highlights** from the second funding period, may be found on page 16-17. This list includes a number of publications, based on fruitful collaboration between the research groups that have participated in CSGB. A detailed description of the research results obtained January 2019 – March 2020 for each of the six work packages WP1-WP6, that form the research plan of the second funding period of CSGB, is given on page 18-29.

The remaining funding period represented **intense international activity** for the CSGB staff, involving the organization of (i) *Workshop on Diffusion MRI and Stochastic Geometry*, (ii) *Workshop on Point Processes in Space, Time and Beyond*, (iii) *15th International Congress for Stereology and Image Analysis*, (iv) *20th Workshop on Stochastic Geometry, Stereology and Image Analysis* and (v) *Workshop on Manifold and Shape Statistics*. More details about the five scientific meetings may be found on page 32-35.

The International Congress for Stereology and Image Analysis took place 27 – 30 May 2019 at Department of Mathematics, Aarhus University, and had about 100 participants from a wide range of disciplines. Five **keynote speakers** gave longer talks and a number of **minisymposia** were arranged. A highlight at the conference was the special keynote lecture, given by the grand-old-man of stereology: Luis M. Cruz-Orive. A PhD competition was organized by the International Society for Stereology and Image Analysis, intended to recognize a PhD thesis in the area of stereology and/or image analysis.

On 27 June 2019, a **follow-up meeting** between CSGB and the Villum Foundation took place. After an introduction by Director of Science at the Villum Foundation, Thomas Bjørnholm, and myself, CSGB senior staff members presented three projects which are part of work packages WP2, WP3 and WP4. The Foundation also met with the junior staff and, later, with the Centre management. The future of CSGB beyond 2020 was discussed.

The last **internal CSGB workshop** took place at Brøndums Hotel, Skagen, 7 – 8 November 2019. A total of eighteen workshops of this type have been arranged during 2010-2020. One of the important purposes of these workshops has been to give junior researchers the opportunity to present their research results in an informal forum.

In the following pages, we inform in more detail our colleagues, the Danish funding partner and the Universities of Aarhus, Aalborg and Copenhagen about the organization, research and other centre activities that took place at CSGB in the remaining funding period.

April 2020
Eva B. Vedel Jensen

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ORGANIZATION AND STAFF

ORGANIZATION AND STAFF

STAFF

Centre for Stochastic Geometry and Advanced Bioimaging (CSGB) unites four Danish research groups

- **AU-math:** Stochastic Geometry Group, Department of Mathematics, AU
- **AU-bio:** Section for Stereology and Microscopy, Department of Clinical Medicine, AU
- **AAU:** Spatial Statistics Group, Department of Mathematical Sciences, AAU
- **KU:** Image Section, Department of Computer Science, KU

The staff consisted in 2019 of 7 professors, 13 associate professors, 1 assistant professor, 5 postdocs and 16 PhD students, see page 40-41 for details.

RESEARCH PLAN

The research is in the second funding period of CSGB (1 April 2015 – 31 March 2020) organized in six work packages, see the diagram below. The principal investigators of the work packages are also shown in the diagram, just below the title of the work packages.

WP1

VALUATION THEORY
Eva B. Vedel Jensen and **Markus Kiderlen**

- Tensor valuations and integral geometry
- Isoperimetric inequalities with tensor constraints
- Uniqueness of measurement functions in integral formulae
- Stereology of tensors

WP2

RANDOM SHAPES
Aasa Feragen and **Stefan Sommer**

- Deformation modelling and statistics of deformations
- Modelling and inference in non-linear spaces
- Applications in diffusion weighted imaging

WP3

SPATIAL AND SPATIO-TEMPORAL POINT PROCESSES
Jesper Møller and **Rasmus P. Waagepetersen**

- Point processes on linear networks
- Point process models and inference for attributed directed graphs
- Determinantal point process modelling
- Sparse models for multivariate spatial point processes
- Inference from quadrat count data

WP4

POINT PROCESSES IN BIOIMAGING
Jens R. Nyengaard and **Ute Hahn**

- Nodule identification and classification from confocal microscopy
- Characterization of moving protein clusters using TIRF imaging
- Synaptic vesicle sampling, distribution and shape from electron microscopy
- Improved variance prediction for stereological estimators of total number

WP5

STATISTICS FOR STOCHASTIC GEOMETRY MODELS
Ute Hahn and **Anders Rønn-Nielsen**

- Asymptotics for excursion sets – extremal properties
- Monte Carlo envelope tests for random closed sets
- Estimation of sample spacing for stochastic processes

WP6

ALGORITHMS
Markus Kiderlen and **François Lauze**

- Digital grey-value stereology
- Shape reconstruction from tensors
- Reconstruction and segmentation of 3D images
- Algorithms for single particle cryo-electron microscopy

NEWS ABOUT STAFF | 1 January 2019 – 31 March 2020

Aasa Feragen (DTU)
In May 2019, Aasa Feragen was appointed full professor at DTU Compute, Department of Applied Mathematics and Computer Science, Technical University of Denmark. In the second funding period of CSGB (2015-2020), Aasa Feragen has been one of the principal investigators of the **Random shapes** work package. She has been a CSGB staff member since 2010 where she started as postdoc. In 2019, she obtained support from the Independent Research Fund Denmark within the thematic research programme on digital technologies.

Tom Dela Haije (KU)
During 2019, Tom Dela Haije has obtained funding from the Villum Foundation Experiment Programme. The title of the project is *Unraveling the Stochastic Geometry of the Human Brain*. He has also been very active in the organization of *Workshop on Diffusion MRI and Stochastic Geometry*, 20 – 24 January 2019, Sandbjerg, see page 32-33. Tom Dela Haije has been a CSGB postdoc, participating in the work package **Random shapes**.

Eva B. Vedel Jensen (AU-math)
In 2019, Eva B. Vedel Jensen became honorary member of the International Society for Stereology and Image Analysis. The election as honorary member took place at the *15th International Congress for Stereology and Image Analysis*, 27 – 30 May 2019, Aarhus. A report on the congress in general may be found on page 34-35. Eva B. Vedel Jensen has been the director of CSGB since 2010.

Jesper Møller (AAU)
Jesper Møller was appointed visiting professor (an honorary position) at Department of Probability and Mathematical Statistics, Charles University, Prague, in March 2020. Jesper Møller has very close connections to the research group around Viktor Beneš. In the period 21 October – 1 November 2019, he gave five talks at the Department on selected topics in spatial statistics. Jesper Møller has been one of the principal investigators of the work package **Spatial and spatio-temporal point processes**.

Stefan Sommer (KU)
Stefan Sommer has from May 2019 received additional funding from a project grant in bioscience and basic biomedicine, supported by the Novo Nordisk Foundation. The title of the project is *Probabilistic Shape Statistics for Disentangling Subtypes in Alzheimer's Disease*. He was also the prime organizer of the *Workshop on Manifold and Shape Stochastics*, 23 – 27 February 2020, Sandbjerg. In the second funding period of CSGB (2015-2020), Stefan Sommer has been a principal investigator of the **Random shapes** work package.

ORGANIZATION AND STAFF

PHD DEFENSES

The following PhD students at CSGB defended their thesis in the period 1 January 2019 – 31 March 2020:






March 2019 | **Line Kühnel** (KU)
Stochastic modelling on manifolds

May 2019 | **Francisco Cuevas** (AAU)
Second order moment properties of point processes and random fields defined on a Euclidean space, a sphere or the product of such spaces

October 2019 | **Helene Svane** (AU-math)
Reconstructing r-regular objects from trinary digital images

November 2019 | **Heidi Søgaard Christensen** (AAU)
Statistics for point processes on linear networks and on the space cross sphere

November 2019 | **Andreas Dyreborg Christoffersen** (AAU)
Iterated and anisotropic marked point processes with a view to the minicolumn hypothesis

January 2020 | **Anton Mallasto** (KU)
Geometric methods in probabilistic modelling

In July 2019, Ninna Vihrs started as PhD student at the AAU group, supported by other sources than CSGB. It is planned that she will participate in a continuation of the minicolumn project.

ORGANIZATION OF INTERNATIONAL WORKSHOPS, CONFERENCES AND PHD COURSES

In the remaining funding period (1 January 2019 – 31 March 2020), the CSGB staff took part in the organization of a number of international workshops, conferences and PhD courses. Some of them are listed on page 32. Below, additional information is given.

Markus Kiderlen (AU-math) obtained substantial additional financial support from Aarhus University Research Foundation for the *20th Workshop on Stochastic Geometry, Stereology and Image Analysis*, 2 – 7 June 2019, Sandbjerg. The workshop had about 70 participants from all over the world, see also page 33.

Christophe Biscio (AAU) obtained in 2019 funding from the Carlsberg Foundation to organize the upcoming *Conference in Data Science Computing*, 4 – 6 August 2020, Aalborg University.

Aasa Feragen (DTU) was program chair at the *International Conference on Medical Imaging with Deep Learning*, 8 – 10 July 2019, London. For further details, see https://2019.midl.io/. She was also a member of the paper selection committee at the *26th International Conference on Information Processing in Medical Imaging (IPMI)*, Hong Kong, 2 – 7 June 2019. Furthermore, she was a member of the program committee of the *Medical Imaging meets NeurIPS Workshop*, 14 December 2019, Vancouver. See also https://sites.google.com/view/med-neurips-2019.

FOLLOW-UP MEETING - 27 JUNE 2019

On 27 June 2019, a follow-up meeting took place at Department of Mathematics, Aarhus University, between CSGB and the Villum Foundation.

From the Villum Foundation, the following participated: Director of Science Thomas Bjørnholm, Professor Anja Boisen (DTU, member of the Foundations's board), Professor Christian S. Jensen (AAU, member of the Foundation's board) and Senior Adviser Michel M.H. Kristensen. From CSGB, 26 staff members participated, representing about 65% of the staff. In addition, head of Department of Mathematics, AU, Jacob Schach Møller participated in the meeting.

After an introduction by Director of Science Thomas Bjørnholm, Eva B. Vedel Jensen gave a brief overview of CSGB, including the present staff, centre activities, status of the work packages and selected highlights since the last follow-up meeting. This overview was followed by the presentation of three projects at CSGB:

- **Markus Kiderlen** (AU-math): *On the precision of stereological volume estimators from planar sections*
- **Rasmus Waagepetersen** (AAU): *Interacting curves in complex environments*
- **Stefan Sommer** (KU): *Geometric statistics: when space curves and estimation becomes smeary*

Afterwards, the Foundation met with the junior staff and, later, with the Centre management. The future of CSGB beyond the present funding period (2015-2020) was discussed.

SELECTED HIGHLIGHTS

- During 2019, a number of papers written by members of the **AAU group** has been published or accepted for publication in absolute top international statistical journals, including *Annals of Statistics and Journal of the American Statistical Association*.
- **Rune Kok Nielsen** (KU) gave an oral presentation at the *International Conference on Medical Image Computing and Computer-Assisted Intervention (MICCAI) 2019*. This conference is the most prestigious conference in medical image analysis. The success rate for oral presentations was 3.3%.
- **Tom Dela Haije** (KU) won a best paper award at *CDMRI 2019*.

NEW COLLABORATIONS

Aasa Feragen (DTU) has established collaboration with Simone Vantini and Anna Calissano, MOX Laboratory for Modeling and Scientific Computing, Politecnico Milano.

Stefan Sommer (KU) started collaboration with Stephan Huckemann and Benjamin Eltzner, Institute for Mathematical Stochastics, Georg-August-Universität, Göttingen, concerning central limit theorems for diffusion means on manifolds.

Jakob G. Rasmussen (AAU) has established collaboration with Leif Sörnmo and Alba Martín-Yebra, Department of Biomedical Engineering, Lund University.

Christophe Biscio (AAU) started in 2019 a collaboration with researchers from INRAE, Avignon, on asymptotic properties of standard estimators in spatial statistics.

SOFTWARE

Rune Kok Nielsen (KU) and coworkers have developed the software package TopAware (https://github.com/RuneKokNielsen/TopAwaRe/). State-of-the-art algorithms for deformable registration restrict to diffeomorphisms to regularize an otherwise ill-posed problem. A novel, piecewise-diffeomorphic deformation framework is presented which models topological changes as explicitly encoded discontinuities in the deformation fields. The entire model is GPU-implemented. See also page 20-21.

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RESEARCH

RESEARCH

In the second funding period of CSGB, the research has been organized in six work packages

WP1: Valuation theory
WP2: Random shapes
WP3: Spatial and spatio-temporal point processes
WP4: Point processes in bioimaging
WP5: Statistics for stochastic geometry models
WP6: Algorithms

A detailed description of the research results obtained in the remaining funding period 1 January 2019 - 31 March 2020 may be found on page 18-29.

Below, we present publication highlights for the full second funding period. The diagramme indicates some of the publications, involving collaboration between the participating research groups.

PUBLICATION HIGHLIGHTS - Second funding period (2015-2020)

Andersen, I.T., Hahn, U., Arnspang, E.C., Nejsum, L.N. & Jensen, E.B.V. (2018): Double Cox cluster processes – with applications to photoactivated localization *microscopy. Spat. Stat.* **27**, 58-73.

Arnaudon, A., Holm, D.D. & Sommer, S. (2019): A geometric framework for stochastic shape analysis. *Found. Comput. Math.* **19**, 653-701.

Baddeley, A., Turner, R. & Rubak, E. (2015): *Spatial Point Patterns: Methodology and Applications with R*. Chapman & Hall/CRC, Boca Raton.

Biscio, C.A.N. & Møller, J. (2019): The accumulated persistence function, a new useful functional summary statistic for topological data analysis, with a view to brain artery trees and spatial point process applications. *J. Comput. Graph. Stat.* **28**, 671-681.

Choiruddin, A., Cuevas-Pacheco, F., Coeurjolly, J.-F. & Waagepetersen, R. (2020): Regularized estimation for highly multivariate log Gaussian Cox processes. *Stat. Comput.*, in press. DOI: 10.1007/s11222-019-09911-y.

Couerjolly, J.-F., Guan, Y., Khanmohammadi, M. & Waagepetersen, R.P. (2016): Towards optimal Takacs-Fiksel estimation. *Spat. Stat.* **18**, 396-411.

Dela Haije, T. & Feragen, A. (2019): Optimized response function estimation for spherical deconvolution. *International MICCAI Workshop on Computational Diffusion MRI.*

Dela Haije, T., Özarslan, E. & Feragen, A. (2020): Enforcing necessary non-negativity constraints for common diffusion MRI models using sum of squares programming. *NeuroImage*, in press. DOI:10.1016/j.neuroimage.2019.116405.

Eriksen, R. & Kiderlen, M. (2020): Uniqueness of the measurement function in Crofton's formula. *Adv. Appl. Math.*, in press. DOI: 10.1016/j.aam.2020.102004.

Guan, Y., Jalilian, A. & Waagepetersen, R. (2015): Quasi-likelihood for spatial point patterns. *J. Roy. Stat. Soc. B* **77**, 677-697.

Hasselholt, S., Hahn, U., Jensen, E.B.V. & Nyengaard, J.R. (2019): Practical implementation of the planar and spatial rotator in a complex tissue: the brain. *J. Microsc.* **273**, 26-35.

Jensen, E.B.V. & Kiderlen, M. (2017, eds.): *Tensor Valuations and their Applications in Stochastic Geometry and Imaging. Lecture Notes in Mathematics* **2177**. Springer.

Kasenberg, N., Darkner, S., Hahn, U., Liptrot, M.G. & Feragen, A. (2016): Structural parcellation of the Thalamus using shortest-path tractography. In: *Proceedings of the International Symposium on Biomedical Imaging (ISBI)*, pp. 559-563.

Khanmohammadi, M., Darkner, S., Nava, N., Nyengaard, J.R., Wegener, G., Popoli, M. & Sporring, J. (2017): 3D analysis of synaptic vesicle density and distribution after acute foot-shock stress by using serial transmission electron microscopy. *J. Microsc.* **265**, 101-110.

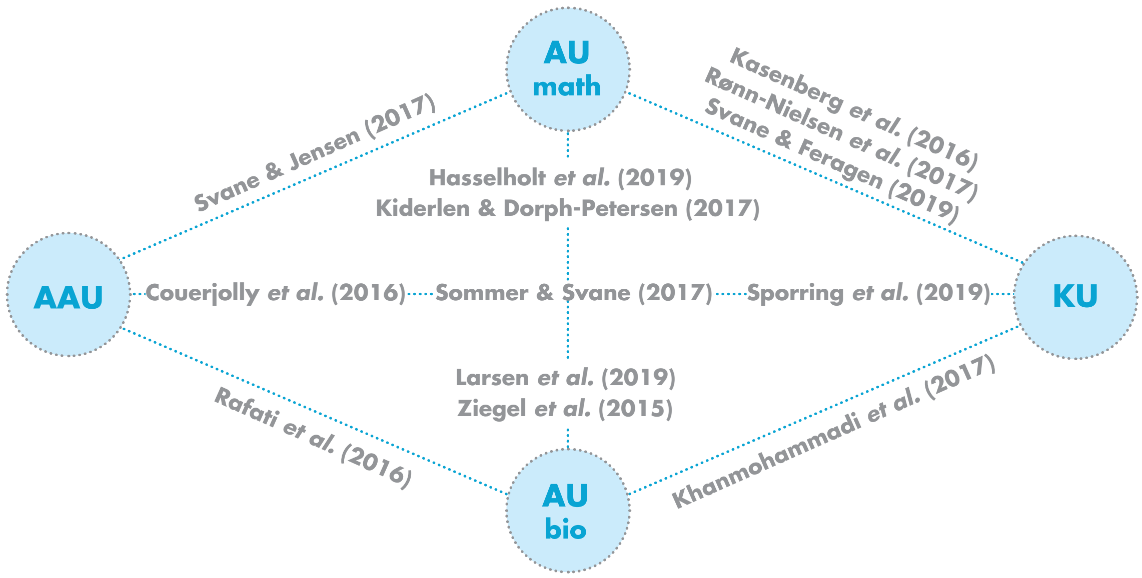
Kiderlen, M. & Dorph-Petersen, K.-A. (2017): The Cavalieri estimator with unequal section spacing revisited. *Image Anal. Stereol.* **36**, 133-139.

Kousholt, A. (2017): Reconstruction of *n*-dimensional convex bodies from surface tensors. *Adv. Appl. Math.* **83**, 115-144.

Kousholt, A. & Kiderlen, M. (2016): Reconstruction of convex bodies from surface tensors. *Adv. Appl. Math.* **76**, 1-33.

Kühnel, L. & Sommer, S. (2017): Stochastic development regression on non-linear manifolds. In: *Proceedings of the International Conference on Image Processing in Medical Imaging (IPMI 2017). Lecture Notes in Computer Science* **10265**, pp. 53-64. Springer.

Larsen, N.Y., Ziegel, J.F., Nyengaard, J.R. & Jensen, E.B.V. (2019): Stereological estimation of particle shape from vertical sections. *J. Microsc.* **275**, 183-194.



Lauze, F., Quéau, Y. & Plenge, E. (2017): Simultaneous reconstruction and segmentation of CT scans with shadowed data. In: *Proceedings of the International Conference on Scale Space and Variational Methods in Computer Vision (SSVM 2017). Lecture Notes in Computer Science* **10302**, pp. 308-319. Springer.

Mallasto, A. & Feragen, A. (2017): Learning from uncertain curves: The 2-Wasserstein metric for Gaussian processes. *Advances in Neural Information Processing Systems* **30** (NIPS 2017), 1-11.

Myllymäki, M., Mrkvička, T., Grabarnik, P., Seijo, H. & Hahn, U. (2017): Global envelope tests for spatial processes. *J. Roy. Stat. Soc. B* **79**, 381-404.

Møller, J. & Christoffersen, A.D. (2018): Pair correlation functions and limiting distributions of iterated cluster point processes. *J. Appl. Probab.* **55**, 789-809.

Møller, J., Nielsen, M., Porcu, E. & Rubak, E. (2018): Determinantal point process models on the sphere. *Bernouilli* **24**, 1171-1201.

Møller, J., Safavimanesh, F. & Rasmussen, J.G. (2016): The cylindrical *K*-function and Poisson line cluster point processes. *Biometrika* **103**, 937-954.

Pennec, X., Sommer, S. & Fletcher, T. (2020, eds.): *Riemannian Geometric Statistics in Medical Image Analysis*. Elsevier. DOI: 10.1016/C2017-0-01561-6.

Rafati, A.H., Safavimanesh, F., Dorph-Petersen, K.-A., Rasmussen, J.G., Møller, J. & Nyengaard, J.R. (2016): Detection and spatial characterization of minicolumnarity in the human cerebral cortex. *J. Microsc.* **261**, 115-126.

Rasmussen, J.G. & Christensen, H.S. (2020): Point processes on directed linear networks. *Methodol. Comput. Appl. Probab.*, in press. DOI: 10.1007/s11009-020-09777-y.

Rønn-Nielsen, A. & Jensen, E.B.V. (2017): Excursion sets of infinitely divisible random fields with convolution equivalent Lévy measure. *J. Appl. Probab.* **54**, 833-851.

Rønn-Nielsen, A., Sporring, J. & Jensen, E.B.V. (2017): Estimation of sample spacing in stochastic processes. *Image Anal. Stereol.* **36**, 43-49.

Sommer, S.H. & Svane, A.M. (2017): Modelling anisotropic covariance using stochastic development and sub-Riemannian frame bundle geometry. *J. Geom. Mech.* **9**, 391-410.

Sporring, J., Waagepetersen, R.P. & Sommer, S.H. (2019): Generalizations of Ripley's *K*-function with application to space curves. In: *Proceedings of the International Conference on Image Processing in Medical Imaging (IPMI 2019). Lecture Notes in Computer Science* **11492**, pp. 731-742. Springer.

Stephensen, H.J.T., Darkner, S. & Sporring, J. (2020): Restoring drifted electron microscope volumes using synaptic vesicles at sub-pixel accuracy. *Commun. Biol.*, in press. DOI: 10.1038/s42003-020-0809-4.

Svane, A.M. (2015): Asymptotic variance of grey-scale surface area estimators. *Adv. Appl. Math.* **62**, 41-73.

Svane, A.M. & Jensen, E.B.V. (2017): Rotational Crofton formulae for Minkowski tensors and some affine counterparts. *Adv. Appl. Math.* **91**, 44-75.

Svane, H. & Feragen, A. (2019): Reconstruction of objects from noisy images at low resolution. In: *Proceedings of the 12th International Workshop on Graph-based Representations in Pattern Recognition. Lecture Notes in Computer Science* **11510**, pp. 204-214. Springer.

Xu, G., Waagepetersen, R. & Guan, Y. (2019): Stochastic quasi-likelihood for case-control point pattern data. *J. Am. Stat. Assoc.* **114**, 631-644.

Ziegel, J.F., Nyengaard, J.R. & Jensen, E.B.V. (2015): Estimating particle orientation and shape using volume tensors. *Scand. J. Stat.* **42**, 813-831.

WP1

Researchers
Johnny B. Andersen
Karl-Anton Dorph-Petersen
Rikke Krog Eriksen
Eva B. Vedel Jensen
Markus Kiderlen
Nick Yin Larsen
Jens R. Nyengaard
Andrew du Plessis
Mads Stehr
Anne Marie Svane
Johanna F. Ziegel

Valuation theory

During the funding period of CSGB (2010-2020), an important focus point has been **tensor valuations and rotational integral geometry**. Motivated by applications in microscopy, rotational Crofton formulae and principal rotational formulae for Minkowski tensors have been developed during this period. Many of these results are reviewed in Jensen & Kiderlen (2017a). See also the overview Jensen & Kiderlen (2017b, eds.) of the modern theory of tensor valuations, presented in the *Springer Lecture Notes in Mathematics Series*. A book project (Jensen, 2020+) has been initiated in 2019 with the purpose to make these research results available to the stochastic geometry community in a broad sense. Applications in microscopy will be included in the planned monograph.

In 2019, CSGB staff members Rikke Krog Eriksen and Markus Kiderlen finalized a comprehensive paper on **uniqueness of measurement functions** (Eriksen & Kiderlen, 2020a). This research was originally motivated by the observation that two apparently very different measurement functions were suggested in local stereology for estimation of surface area. In Eriksen & Kiderlen (2020a), the question of uniqueness of measurement functions in **Crofton's formula** is investigated. Crofton's formula states for compact convex subsets K in $\mathbb{R}^n$ that

$$\int_{A(n,k)} V_j(K \cap E)\, \mu_k(\mathrm{d}E) = \alpha_{n,j,k} V_{n+j-k}(K),$$

$0 \leq j \leq k \leq n-1$, where $A(n,k)$ is the space of k-dimensional affine subspaces of $\mathbb{R}^n$, V_j is the jth intrinsic volume, μ_k is the motion invariant measure on $A(n,k)$ and $\alpha_{n,j,k}$ is a known constant. The question is now whether there exist other functionals φ (possibly under additional restrictions on φ) for which

$$\int_{A(n,k)} \varphi(K \cap E)\, \mu_k(\mathrm{d}E) \propto V_{n+j-k}(K). \quad (1)$$

It turns out that this easily posed question has a rather involved answer.

In Eriksen & Kiderlen (2020a), the focus is on the subclass of **local functionals** φ. The local property is formally defined in Weil (2017). An extension of Weil (2017, Theorem 2.1) to local functionals on compact convex sets of dimension at most $k \leq n-1$ is shown in Eriksen & Kiderlen (2020a) and a decomposition of such functionals into homogeneous local functionals is given. The conclusion is that the measurement function is not unique, but within the class of local functionals a characterization of the solutions to (1) can be given for $k = 1$. Likewise, for $k = 2$, a characterization is available for φ amongst even local functionals. Additional functionals are constructed for $k > 2$.

The recent research on **stereology of tensors** has been published in Larsen *et al.* (2019). In this paper, a simple local stereological method of estimating particle shape from vertical sections is presented to scientists working in microscopy. The method uses measurements in a vertical optical plane, passing through a reference point of each sampled particle. The measurements performed on a sampled particle are illustrated in Figure 1. A systematic set of alternating half lines (indexed by i) is used on each sampled particle profile, generated by the vertical plane. If we let l_{ij} be the distance from the jth intersection point on the ith half line to the vertical axis, see Figure 1, it is needed to determine the so-called squared ray distance for the ith half line

$$l_i^2 = \sum_{j \text{ even}} l_{ij}^2 - \sum_{j \text{ odd}} l_{ij}^2.$$

Likewise, the power-4 ray distance is determined

$$l_i^4 = \sum_{j \text{ even}} l_{ij}^4 - \sum_{j \text{ odd}} l_{ij}^4.$$

The estimator of particle shape is based on estimators of volume tensors of the sampled particle which are simple functions of the following quantities

$$\sum_i l_i^2, \sum_i z_i l_i^2, \sum_i l_i^4, \sum_i z_i^2 l_i^2.$$

Here, z_i is the (signed) distance from the reference point (denoted O in Figure 1) to the ith half line.

The method is superiour to earlier, more time-consuming methods, based on observation in several optical planes. In a model-based setting, the new method requires that the particle distribution is invariant under rotations around the vertical axis. For a detailed discussion of this assumption, see Hasselholt *et al.* (2019). As a new contribution, a dual design-based approach is developed in the paper Larsen *et al.* (2019). In the design-based case, it is not needed to assume **restricted isotropy**, and the method provides information about an index of elongation of the particles in the direction of the vertical axis. We have also managed to generalize the method to n–dimensional space, see the forthcoming publication Eriksen & Kiderlen (2020b). A review, with emphasize on the importance of vertical sections in stereology, may be found in Jensen (2020).

The research Bianchi *et al.* (2019) on mappings that can be characterized as rearrangements, in particular polarizations, was also finalized in 2019.

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Eriksen, R. & Kiderlen, M. (2020b): Volume and surface tensors of particle processes under *L*-restricted isotropy. In preparation.

Hasselholt, S., Hahn, U., Jensen, E.B.V. & Nyengaard, J.R. (2019): Practical implementation of the planar and spatial rotator in a complex tissue: the brain. *J. Microsc.* **273**, 26-35.

Jensen, E.B.V. (2020): Stereological inference on particle shape from vertical sections. *CSGB Research Report* **2020-01**. Submitted.

Jensen, E.B.V. (2020+): *Rotational Integral Geometry and its Applications.* In preparation.

Jensen, E.B.V. & Kiderlen, M. (2017a): Rotation invariant valuations. In: *Tensor Valuations and their Applications in Stochastic Geometry and Imaging* (eds. E.B.V. Jensen and M. Kiderlen). *Lecture Notes in Mathematics* **2177**, pp. 185-212. Springer.

Jensen, E.B.V. & Kiderlen, M. (2017b, eds.): *Tensor Valuations and their Applications in Stochastic Geometry and Imaging. Lecture Notes in Mathematics* **2177**. Springer.

Larsen, N.Y., Ziegel, J.F., Nyengaard, J.R. & Jensen, E.B.V. (2019): Stereological estimation of particle shape from vertical sections. *J. Microsc.* **275**, 183-194.

Weil, W. (2017): Integral geometry of translation invariant functionals, II: The case of general convex bodies. *Adv. Appl. Math.* **83**, 145-171.

i	l_{i0}	l_{i1}	l_{i2}	z_i	l_i^2	$z_i l_i^2$	$z_i^2 l_i^2$	l_i^4
1	2.1	1.4	0.9	1.4	3.26	4.564	6.3896	16.2626
2	1.4			0.2	1.96	0.392	0.0784	3.8416
3	1.2			-1.0	1.44	-1.440	1.4400	2.0736
4	1.2	0.1		-2.2	1.43	-3.146	6.9212	2.0735
Σ					8.09	0.370	14.8292	24.2513

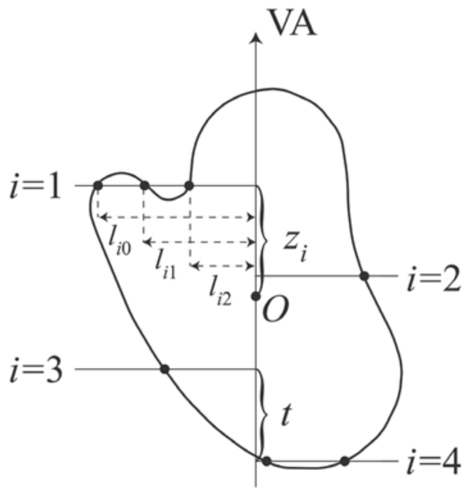

Figure 1
Illustration of the measurements on a sampled particle profile. A systematic set of alternating half lines, perpendicular to the vertical axis (VA), is used. The intersection points on a given half line are numbered according to decreasing distance to VA, using number 0 for the most distant intersection point. We let l_{ij} be the distance from the jth intersection point on the ith half line to VA and z_i is the (signed) distance from the reference point (denoted O) to the ith half line. The measurements in the table are in arbitrary units and refer to the shown particle profile. The distance between neighbour half lines is $t = 1.2$. For details, see Larsen *et al.* (2019).

WP2

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Random shapes

For data taking values in **non-linear spaces**, a number of advances were made in the remaining funding period. Wrapped Gaussian Processes (WGPs) were utilized to define probabilistic submanifold learning algorithms (Mallasto *et al.*, 2019). More precisely, when data is known to reside on a manifold, for instance due to invariances or constraints, a WGP latent variable model is used in Mallasto *et al.* (2019) to learn a stochastic data manifold, which is ensured to satisfy the constraints or invariances by being constrained to lie within the manifold. Beyond the realm of manifolds, we started collaborating with Anna Calissano and Simone Vantini at the MOX Laboratory at Politecnico di Milano on non-linear statistics for network data in mobility planning.

Geometry also enters into statistical modelling through geometric properties of the models themselves, a fact that has been used in several subprojects. State-of-the-art image registration assumes, largely for the sake of regularization, that all anatomies are diffeomorphic **deformations** of a standard anatomy. However, anatomy varies even in healthy subjects. In diseased subjects, matter appears in the form of tumors, bleeds and cavities. It disappears e.g. via surgery or atrophy. We have made the first steps towards solving these problems by introducing piecewise diffeomorphic models that allow for the introduction or removal of matter (Kok Nielsen *et al.*, 2019). This paper was awarded an oral presentation at the highest ranking conference in medical imaging, MICCAI, with an acceptance rate of 3.3%.

In Dela Haije *et al.* (2020), constrained optimization is utilized to ensure that local microstructural models estimated from **diffusion magnetic resonance images** (dMRI) actually satisfy known geometric constraints. This is hugely important. State-of-the-art microstructural models are shown to defy natural laws in large portions of the brain, potentially affecting down-stream analysis of both microstructure and connectivity.

Also related to local microstructural models in dMRI, Dela Haije & Feragen (2019) use rotational invariants to obtain unbiased estimates of the so-called response functions in constrained spherical deconvolution. This paper also won the best paper award.

In Jensen *et al.* (2019), an information-theoretic approach to **registration of diffusion weighted images** with explicit optimization over the orientational scale is presented. The LOR-DWI density-based hierarchical scale-space model is extended to non-rigid deformations, and it is shown that the formulation provides intrinsic regularization through the orientational information.

When the data are elements of non-linear spaces, such as differentiable manifolds, methodology for **simulating bridge processes** is lacking. In particular, in cases where the transition probability densities are intractable, it is of interest to use simulation schemes that can numerically approximate the true densities. In Højgaard Jensen *et al.* (2019), a method for simulating diffusion bridges on the flat torus is proposed. This specific case serves as an example of the more general setting of simulating diffusion bridge processes on Riemannian manifolds.

Frank van der Meulen from TU Delft spent a six month sabbatical in Copenhagen working on **bridge sampling** on non-linear spaces. The work has resulted in successful transfer of sampling techniques from the Euclidean setting to examples of manifold valued data. In particular, the developed scheme covers stochastic models for landmark manifolds for which no prior sampling scheme has been available.

Data is often observed without ordering implying that a sample of multiple observations are naturally modelled in configuration spaces, the *n*-fold product modulo permutations. When the data space itself is a manifold, this quotient space has interesting geometric structure. Together with Philipp Harms, Xavier Pennec and Peter Michor, we have explored geometric statistics on configuration spaces, including proving a **central limit theorem for unordered samples** without the common assumption of independence between the samples.

Many aspects of the work on geometric statistics, performed by the KU group associated with CSGB, has been described in the book Pennec *et al.* (2020) on *Riemannian Geometric Statistics in Medical Image Analysis*, edited by Stefan Sommer together with Xavier Pennec and Tom Fletcher. The book contains multiple contributions from the KU group.

During 2019, the research on a geometric framework for stochastic shape analysis (Arnaudon *et al.*, 2019), deterministic group tractography (Holm *et al.*, 2019), deep learning numerics (Kühnel *et al.*, 2019) and principal component analysis on manifolds (Sommer, 2019) has been published.

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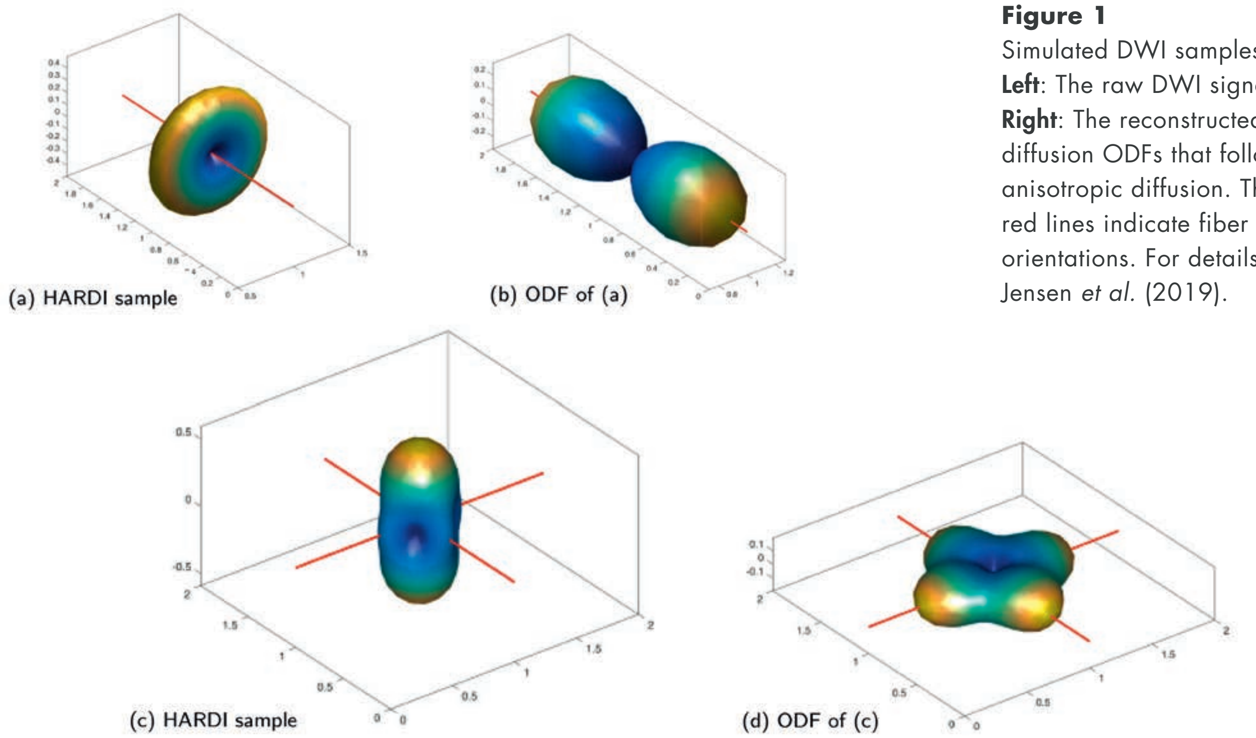

Figure 1
Simulated DWI samples. **Left**: The raw DWI signal. **Right**: The reconstructed diffusion ODFs that follow anisotropic diffusion. The red lines indicate fiber orientations. For details, see Jensen *et al.* (2019).

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Spatial and spatio-temporal point processes

A highlight was the acceptance in the very top journal *Annals of Statistics* of the 35 pages long paper by CSGB researchers Jesper Møller and Jakob Gulddahl Rasmussen (in collaboration with Ethan Anderes) on isotropic covariance functions on **linear networks** and their extension to **graphs** with Euclidean edges (Anderes *et al.*, 2019). The covariance functions are isotropic in the sense that they only depend on a new metric that extends the classical resistance metric on the vertices of a weighted graph to both the vertices and edges of any graph with Euclidean edges.

The research on the use of **persistence diagrams** for analyzing point patterns resulted in two publications. The diagram captures geometric structures such as point clusters and voids. For this reason, the accumulated persistence function has been suggested in Biscio & Møller (2019) as a functional summary statistics for point patterns. While the geometric robustness of the persistence diagram is well-studied, only limited results on its statistical properties are available. In a recent paper (Biscio *et al.*, 2019), CSGB researchers Christophe Biscio and Anne Marie Svane, together with Christian Hirsch, have studied the use of **topological data analysis** for assessing the goodness of fit of point process models. In particular, they derive a functional central limit theorem for the persistence diagram of a planar point pattern, which allows testing the goodness of fit of a given point process model, using the accumulated persistence function. In a future research project, they seek to extend the results to marked point processes.

Rasmus Waagepetersen and collaborators finished and published in 2019 a paper on second-order variational **estimating equations** for spatial point processes (Couerjolly *et al.*, 2019). A focus point is here pair correlation function estimation. In the case of log linear parametric models for pair correlation functions, it is demonstrated that the variational equations can be applied to construct estimating equations with closed form solutions for the parameter estimates. See also Jalilian *et al.* (2019).

A new method for analysis of **multivariate point process** data is suggested in Hessellund *et al.* (2019). Semi-parametric models for the intensity functions are proposed. A multinomial conditional composite likelihood function is introduced for estimation of intensity function regression parameters and the asymptotic joint distribution of the resulting estimators is derived under mild conditions. The asymptotic covariance matrix depends on the cross pair correlation functions of the multivariate point process. Standardized residual plots and predictive probability plots are constructed for the validation of the model.

Rasmus Waagepetersen and coworkers have also published papers on regularized inference for highly multivariate point processes (Choiruddin *et al.*, 2020), inference for stochastic radio channel models (Hirsch *et al.*, 2020) and adaptive estimating functions for **determinantal point processes** (Lavancier *et al.*, 2019).

The research on leverage and influence diagnostics (Baddeley *et al.*, 2019), α-mixing point processes (Biscio & Waagepetersen, 2019), Voronoi intensity estimators (Mehdi Moradi *et al.*, 2019), *K*-functions for space curves (Sporring *et al.*, 2019) and stochastic quasi-likelihood (Xu *et al.*, 2019) has been published or accepted for publication. This research has been described in *CSGB Annual Report* **2018**. Dvořák *et al.* (2019) contains a quick inference method for log Gaussian Cox processes.

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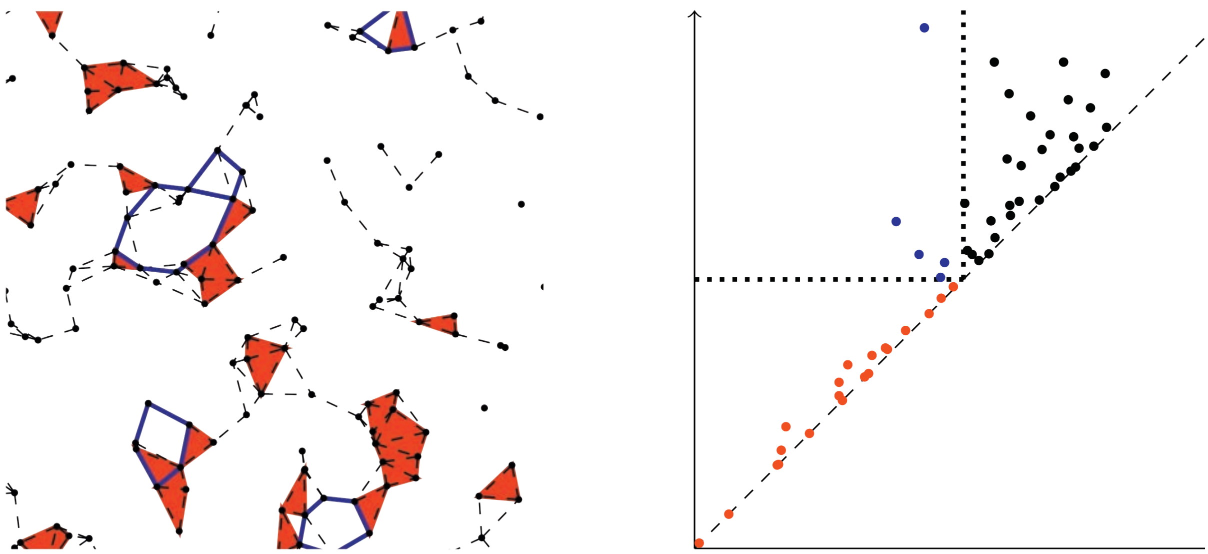

Figure 1
Alpha-complex with alive (blue) and dead (red) loops marked (left) and associated persistence diagram (right). For details, see Biscio *et al.* (2019).

WP4

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Point processes in bioimaging

The project on **improved variance prediction in stereology** has been finalized by CSGB PhD student Mads Stehr and Markus Kiderlen (Stehr & Kiderlen, 2019). This research was originally motivated by the observation that the variance of Cavalieri type stereological estimators may be severely inflated if the sampling points are unequally spaced, see Ziegel *et al.* (2010) and references therein. The results obtained in Stehr & Kiderlen (2019) apply to numerical integration in general when sampling nodes are random. It is suggested to use Newton-Cotes quadrature to exploit the smoothness properties of the integrand. Under some integrability conditions on the typical point-distance, it is shown that Newton-Cotes quadratures based on a stationary point process on $\mathbb{R}$ yield unbiased estimators for the integral and that the aforementioned variance inflation can be avoided if a Newton-Cotes quadrature of sufficiently high order is applied.

CSGB PhD student Heidi Søgaard Christensen together with Jesper Møller completed in 2019 their work on modelling spine locations on dendrite trees, using inhomogeneous Cox point processes (Christensen & Møller, 2019). Dendritic spines are small protrusions on the dendrites of a neuron. In neuroscience, the spatial distribution of the spines on dendrite trees is of interest, as changes in this distribution may be linked to changes in cognitive processes. The spine locations can be viewed as a point pattern on the dendrite tree and thus analyzed, using point process theory for **linear networks**.

There are only a limited number of point process models available for linear networks. An exception is directed acyclic linear networks, for which both regular and clustered models can be defined by a generalization of the conditional intensity function for temporal point processes (Rasmussen & Christensen, 2020). In Christensen & Møller (2019), a new class of Cox process models on a linear network is suggested. Moreover, the use of minimum contrast and composite likelihood procedures is demonstrated, and new empirical summary functions are introduced. The distribution of spine locations on six different dendrite trees from mouse neurons is analyzed, using the new methodology.

The **minicolumn hypothesis** in neuroscience claims that neurons and other brain cells have a columnar arrangement, perpendicular to the surface of the brain. This hypothesis has been extensively studied in the biological literature, see e.g. Buxhoeveden & Casanova (2002), Rafati *et al.* (2016) and references therein. In 2019, Heidi Søgaard Christensen together with Jesper Møller and CSGB PhD student Andreas Dyreborg Christoffersen submitted a paper on modelling columnarity of pyramidal cells in the human cerebral cortex (Christoffersen *et al.*, 2019). In the paper, a new **hierarchical point process model** is suggested, consisting first of a generalized shot noise Cox process in the *xy*-plane, providing cylindrical clusters, and next of a Markov random field model on the *z*-axis. Several cases of these hierarchical point processes are fitted to two pyramidal cell datasets. The location of a pyramidal cell is represented by its nucleolus. The data also includes the local orientation of the cell, determined by the direction from the nucleolus to the apical dendrite's position. However, since it was found in Møller *et al.* (2019) that locations and orientations of the cells were independent, the focus of Christoffersen *et al.* (2019) is on modelling the locations. The new model provides a better fit to these datasets than the so-called Poisson line cluster point process model, earlier suggested in Møller *et al.* (2016).

New spatio-temporal point process models for **super resolution fluorescence microscopy** data have been developed in the remaining funding period. The models account for the blinking characteristics of fluorescent proteins, and analysis tools have been developed that allow for estimation of both model and nuisance parameters, including the fraction of noise points and number of re-apperances per fluorescent protein.

In 2019, the research involving photoactivated localization microscopy was published in Arnspang *et al.* (2019). In continuation of this research, an analysis of renal aquaporin-2 (AQP2) in collecting duct principal cells has been carried out as part of a collaborative research project. The data consists of noisy maps of protein locations, observed in 3D.

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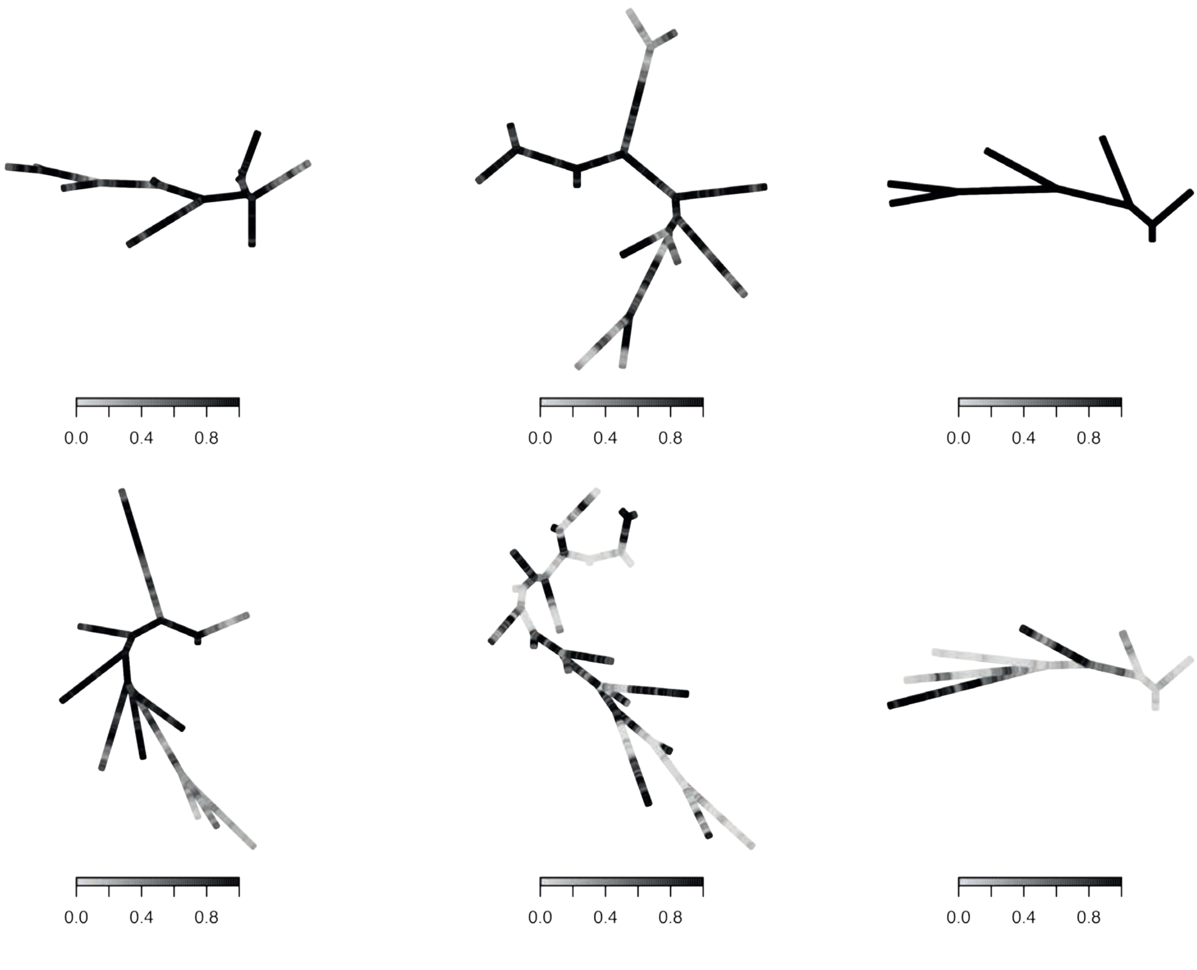

Figure 1
For each dendrite tree, a simulated realization of the random field, determining the retention probabilities in the fitted Cox process models, is shown. For details, see Christensen & Møller (2019).

WP5

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Statistics for stochastic geometry models

The general aim of this work package has been to develop new statistical inference procedures for stochastic geometry models. The focus has been on **Monte Carlo envelope tests** for spatial models, on **asymptotics for excursion sets** and on **estimation of sample spacing**.

During the second funding period of CSGB (2015-2020), three papers on **envelope testing** have been published by CSGB researcher Ute Hahn and collaborators, see Mrkvička *et al.* (2016, 2017) and Myllymäki *et al.* (2017). The paper Myllymäki *et al.* (2017) in the prestigious *Journal of the Royal Statistical Society B* has already received 112 citations, according to Google. The envelope method can be extended for comparison of groups of functional data.

Asymptotics of excursion sets has been studied for Lévy-based random fields. In 2019, the CSGB group (Stehr & Rønn-Nielsen, 2019) has completed a study of Lévy-based models depending on both space and time, generalizing the models appearing in Rønn-Nielsen & Jensen (2016, 2017). The space-time random field

$$X = (X_{v,t})_{(v,t)\in B\times[0,T]}$$

is the Lévy-driven moving average model defined by

$$X_{v,t} = \int_{\mathbb{R}^d\times(-\infty,t]} f(|v-u|, t-s)\, M(\mathrm{d}u,\mathrm{d}s),$$

where M is an infinitely divisible, independently scattered random measure on $\mathbb{R}^{d+1}$, f is some kernel function, and B and $[0,T]$ are compact index sets. The index v refers to the spatial position, while t is interpreted as time. The random field is a causal model since $X_{v,t}$ only depends on the restriction of M to $\mathbb{R}^d \times (-\infty, t]$.

Such Lévy-driven moving average models provide a flexible and tractable modelling framework. Examples of applications, that include both time and space, are modelling of turbulent flows (Barndorff-Nielsen & Schmiegel, 2004) and growth processes (Jónsdóttir *et al.*, 2008). Purely spatial models have been used for modelling of Cox point processes (Hellmund *et al.*, 2008) and brain imaging data (Jónsdóttir *et al.*, 2013; Rønn-Nielsen *et al.*, 2017). The latter paper deals with **estimation of sample spacing**. In Rønn-Nielsen & Jensen (2019), central limit theorems are proved for mean and variogram estimators in Lévy-driven moving average models.

In Stehr & Rønn-Nielsen (2019), it is assumed that the Lévy measure ρ of the random measure M has a convolution equivalent right tail. Under regularity conditions, it is shown that certain functionals of the field X have a right tail that is equivalent to the tail of the underlying Lévy measure, i.e. there exist known constants C and c such that

$$P(\Psi(X) > x) \sim C\rho((x/c, \infty)),$$

as $x \to \infty$. The simplest example of a functional Ψ considered is $\Psi(X) = \sup_{v,t} X_{v,t}$. Another example relates to the spatial excursion set at level x and time t

$$A_{x,t} = \{v \in B : X_{v,t} > x\}.$$

It is shown under further regularity conditions that the asymptotic probability that there exists a time point t for which the excursion set at level x contains a ball of fixed radius has a tail that is equivalent with the tail of ρ. In a third example, a similar result is shown for the probability that an average of the field over a time interval and a set of spatial positions exceeds the level x.

The results obtained in Stehr & Rønn-Nielsen (2019) are generalizations of those obtained in the earlier papers Rønn-Nielsen & Jensen (2016, 2017) for spatial random fields. In Fasen (2008), results for a moving average process on $\mathbb{R}$, obtained as an integral with respect to a Lévy process with convolution equivalent tail, are derived. Here, the process $(X_t)_{t\in[0,T]}$ is given by

$$X_t = \int_{-\infty}^{t} f(t-s)\, M(\mathrm{d}s),$$

where M is a convolution equivalent Lévy measure on $\mathbb{R}$. In agreement with the results in Stehr & Rønn-Nielsen (2019), it is derived in Fasen (2008) that $\sup_t X_t$ has a tail that is asymptotically equivalent to the tail of the underlying Lévy measure.

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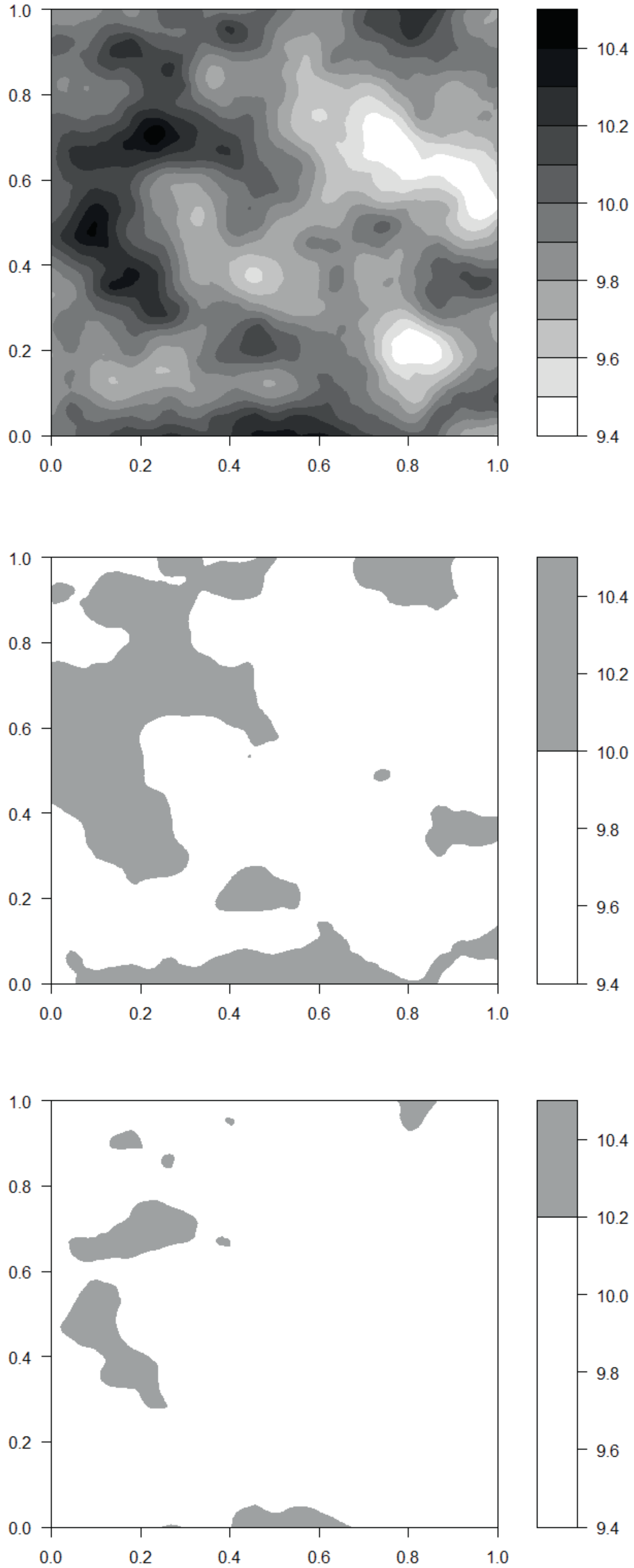

Figure 1
Two excursion sets (middle and lower) for the spatial random field shown in the upper plot. In Stehr & Rønn-Nielsen (2019), excursion sets for space-time random fields are studied.

WP6

Researchers
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Algorithms

In the remaining funding period of CSGB, the focus has been on **reconstruction and segmentation** of objects from digital grey-value images.

In the PhD thesis Svane (2019), reconstruction of both 2D and 3D objects from their digital images are considered. When working with reconstruction from digital images, it is necessary to put some constraints on the objects, being considered, because the digitization process removes features that are small compared to the pixel (voxel) size. In Svane (2019), the objects are assumed to be r-regular. Intuitively, the notion of ***r*-regularity** ensures that an *r*-regular object cannot have a boundary with too many curlicues, or have sections with too narrow necks. A number of authors have formulated such regularity conditions in slightly different ways (Federer, 1959; Serra, 1983; Stelldinger & Köthe, 2005; Duarte & Torres, 2015).

The digital images are constructed by placing the r-regular object on a grid and then colouring the grid squares (or cubes, when we work with 3D images) black if they are completely inside the object, white if they are completely outside the object, and grey otherwise. In this way, a digital image is obtained with just three colours: black, white and grey. Such an image is called a **trinary image**.

The paper Svane & du Plessis (2019), which is part of the PhD thesis Svane (2019) and also available at arXiv, deals with **reconstruction of 2D *r*-regular objects** from their trinary images. The main result of this paper is the following. If we let *d* be the pixel side length, it is possible for an r-regular object X in 2D with $d\sqrt{2} < r$ to construct an object such that the boundaries of the original object and the reconstructed object are closer than d apart in Hausdorff distance. In Figure 1 and 2, the reconstruction of a 2D object from a trinary image is illustrated. Whenever two grey pixels share an edge, an auxiliary point is placed on that edge, see Figure 1, left. Then, a unique circle arc is determined through each three consecutive auxiliary points, see Figure 1, right. Each set of two consecutive auxiliary points is thereby connected by two circle arcs. Interpolating between these two curves for every set of two consecutive auxiliary points, a continuous curve is constructed, forming the boundary of the reconstructed object, see Figure 2.

In the paper Svane & Feragen (2019), which is also part of the PhD thesis Svane (2019), **reconstruction of objects from their noisy images** at low resolution is considered. A method is proposed for removing noise by comparing the observed 3×3 pixel configurations in the noisy image to the 3×3 trinary image configurations that may occur in an ideal digital image of an r-regular object. The solution provided is a globally optimal algorithm that is computationally very demanding. For this reason, a local approach for reconstructing the trinary image is also proposed in Svane & Feragen (2019).

A 2D and 3D variational segmentation approach based on similarity invariant, i.e. translation, scaling and rotation invariant, shape regularisers is presented in Hansen & Lauze (2019). Shape moments of order up to 2 for shapes with limited symmetries can be combined to provide a shape normalization for the group of similarities. In order to obtain a segmentation objective function, a two-means or two-local-means is added to it. Segmentation is then obtained by standard gradient descent. The capabilities of the approach is demonstrated on a series of experiments.

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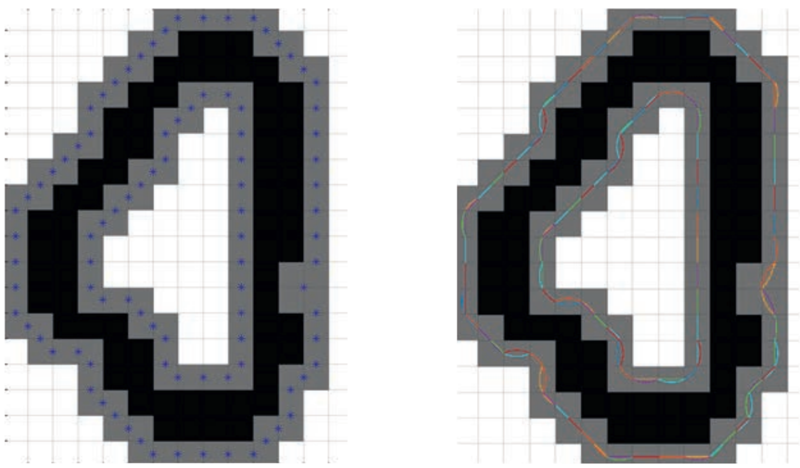

Figure 1
A reconstruction of a trinary image is obtained by first plotting auxiliary points on the boundaries of the grey pixels (left) and then connecting three consecutive auxiliary points by circle arcs (right).

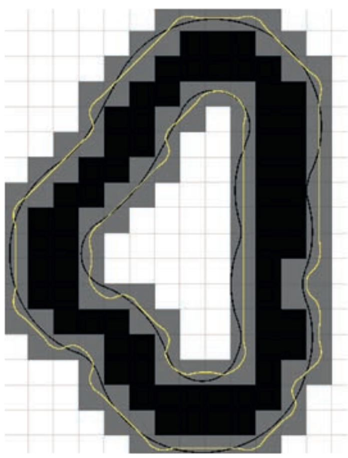

Figure 2
Smooth curves (yellow) are obtained by interpolation of the circular arcs, shown in Figure 1, right. The black curves are the boundaries of the original object.

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CENTRE ACTIVITIES

OVERVIEW

PAST AND PLANNED INTERNATIONAL ACTIVITIES

INTERNATIONAL CONFERENCES, SYMPOSIA AND WORKSHOPS

- *Workshop on Diffusion MRI and Stochastic Geometry*
  20 – 24 January 2019, Sandbjerg
- *Workshop on Point Processes in Space, Time and Beyond*
  13 – 16 May 2019, Skagen
- *15th International Congress for Stereology and Image Analysis*
  27 – 30 May 2019, Aarhus
- *20th Workshop on Stochastic Geometry, Stereology and Image Analysis*
  2 – 7 June 2019, Sandbjerg
- *Workshop on Quantitative Microscopy*
  12 – 16 August 2019, Bern
- *Workshop on Manifold and Shape Stochastics*
  23 – 27 February 2020, Sandbjerg

INTERNATIONAL PHD AND MASTER COURSES

- *Quantitative Medical Graphics*
  27 February 2019, Aarhus
- *Interdisciplinary Summer School on Neuroimaging*
  16 – 26 July 2019, Aarhus
- *PhD Summer School on Generative Models*
  12 – 16 August 2019, Blokhus
- *PhD Course on Design and Analysis of Experiments*
  10 – 24 October 2019, Aalborg
- *PhD Course on Selected Topics in Spatial Statistics*
  21 October – 1 November 2019, Prague
- *PhD Course in Neuroscience*
  6 – 10 November 2019, Beijing
- *PhD Course on Bayesian Statistics, Simulation and Software*
  4 – 11 December 2019, Aalborg

From *Workshop on Diffusion MRI and Stochastic Geometry*, 20 – 24 January 2019, Sandbjerg.
Photo Ute Hahn.

WORKSHOP ON DIFFUSION MRI AND STOCHASTIC GEOMETRY

20 – 24 JANUARY 2019, SANDBJERG ESTATE, DENMARK

Photo Ute Hahn.

Scope of the workshop

The focus of this workshop was to strengthen the connections between diffusion MRI and stochastic geometry. Stochastic geometry is widely used in a variety of fields, including astronomy, geology and telecommunications, but its application in diffusion MRI has so far remained limited. There is a great potential in enhancing the use of stochastic geometry tools in diffusion MRI, where different types of challenging random geometrical patterns are observed. At the workshop, leading researchers from the two communities were brought together to discuss recent advances and connections between the fields.

Structure of the workshop

The workshop had longer keynote talks and shorter invited talks, as well as a poster session.

Organizers

The workshop was organized by CSGB staff members:

- Tom Dela Haije (KU)
- Aasa Feragen (DTU)
- Eva B. Vedel Jensen (AU-math)
- Oddbjørg Wethelund (AU-math)

20TH WORKSHOP ON STOCHASTIC GEOMETRY, STEREOLOGY AND IMAGE ANALYSIS

2 – 7 JUNE 2019, SANDBJERG ESTATE, DENMARK

Photo Ute Hahn.

Organizers and financial support

The organizers of this workshop were CSGB staff members from AU-math:

- Ute Hahn
- Eva B. Vedel Jensen
- Markus Kiderlen
- Oddbjørg Wethelund

Financial support was obtained from the Aarhus University Research Foundation.

Scope of the workshop

This workshop was the 20th in a biennial series of meetings, bringing together researchers and practitioners working with random geometric objects. The goal of the workshop was to promote advances in all branches of stochastic geometry and related fields including, but not limited to, the theory of point processes, integral geometry, stereology, discrete and continuum percolation and random graphs, spatial statistics, bioimaging and all other application areas of spatial stochastic modelling.

Structure of the workshop

The workshop had longer talks by invited speakers and shorter contributed talks by the participants, as well as a poster session.

15TH INTERNATIONAL CONGRESS FOR STEREOLOGY AND IMAGE ANALYSIS

27 – 30 MAY 2019, AARHUS UNIVERSITY, DENMARK

The 15th International Congress for Stereology and Image Analysis took place 27 – 30 May 2019 at Department of Mathematics, Aarhus University, Denmark (http://conferences.au.dk/icsia2019/).

On Monday morning, the organizing staff welcomed about 100 researchers from a wide range of disciplines and Jens R. Nyengaard opened the conference.

Photos by CSGB staff.

SCIENTIFIC PROGRAMME

The programme of the conference was organized in two parallel sessions.
Five keynote speakers gave longer talks:

Michael Klatt
(Princeton)

Mari Myllymäki
(Helsinki)

Emilio Porcu
(Newcastle)

Katja Schladitz
(Kaiserslautern)

Fei Sun
(Beijing)

In addition, a number of minisymposia were arranged (organizer(s) in parenthesis):

- *Advanced quantification of lung tissue* (Matthias Ochs and Christian Mühlfeld)
- *Digital pathology* (Torben Steiniche)
- *Fibre processes and orientation maps* (Jon Sporring)
- *Image analysis and stochastic modelling of materials* (Claudia Redenbach)
- *Machine learning techniques for image segmentation and object detection* (Volker Schmidt)
- *Mathematical morphology in the deep learning era* (Jesus Angulo and Santiago Velasco-Forero)
- *Neurostereology and image analysis* (Dorothy Oorschot and Jens R. Nyengaard)
- *Sampling in stereology* (Marcos Cruz)
- *Stochastic modelling of 3D microstructures for virtual materials testing* (Volker Schmidt)

The International Society for Stereology and Image Analysis organized a PhD competition intended to recognize a PhD thesis conducted in the area of stereology and/or image analysis. **Johannes Österreicher** won this competition and gave a lecture about his PhD thesis in plenum.

A highlight at the conference was the special keynote lecture, given by the grand-old-man of stereology: **Luis M. Cruz-Orive**. The title of the lecture was *Stereology – personal recollections*.

A poster session was also part of the conference programme.

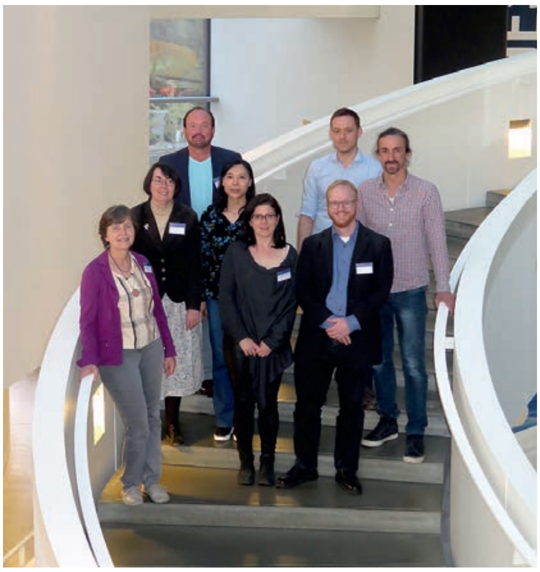
SOCIAL EVENTS

The welcome reception on Monday 27 May took place at the Greenhouses in the Botanical Garden, Aarhus. A second group photo was taken here.

A conference dinner was arranged at the art museum in Aarhus, ARoS. Prior to the dinner, it was possible to participate in a guided tour at ARoS. Leading members of the International Society for Stereology and Image Analysis were photographed at ARoS.

Organization

The conference was organized by Ute Hahn, Eva B. Vedel Jensen, Markus Kiderlen, Jens R. Nyengaard and Oddbjørg Wethelund.



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EIGHTEENTH INTERNAL CSGB WORKSHOP

Brøndums Hotel, Skagen, 7 – 8 November 2019

The last internal CSGB workshop took place at Brøndums Hotel, Skagen, 7 – 8 November 2019. During the full funding period 2010-2020 of CSGB, these workshops have played an important role. The present status of the CSGB research projects has been discussed at these workshops and plans for further progress have been made. Furthermore, the junior researchers have had the opportunity to present their research results in an informal forum.

Photo Pernille Hansen.

CSGB RESEARCH REPORTS

1 January 2019 – 31 March 2020

1. Bianchi, G., Gardner, R.J., Gronchi, P. & Kiderlen, M. (2019): Rearrangement and polarization. *CSGB Research Report* **2019-12**. Submitted.

2. Biscio, C.A.N., Chenavier, N., Hirsch, C. & Svane, A.M. (2019): Testing goodness of fit for point processes via topological data analysis. *CSGB Research Report* **2019-11**. Submitted.

3. Choiruddin, A., Coeurjolly, J.-F. & Waagepetersen, R. (2020): Information criteria for inhomogeneous spatial point processes. *CSGB Research Report* **2020-04**.

4. Choiruddin, A., Cuevas-Pacheco, F., Coeurjolly, J.-F. & Waagepetersen, R. (2020): Regularized estimation for highly multivariate log Gaussian Cox processes. *CSGB Research Report* **2020-03**. *Stat. Comput.*, in press.

5. Christensen, H.S. & Møller, J. (2019): Modelling spine locations on dendrite trees using inhomogeneous Cox point processes. *CSGB Research Report* **2019-06**. Submitted.

6. Christoffersen, A.D., Møller, J. & Christensen, H.S. (2019). Modelling columnarity of pyramidal cells in the human cerebral cortex. *CSGB Research Report* **2019-08**. Submitted.

7. Coeurjolly, J.-F., Cuevas-Pacheco, F. & Waagepetersen, R. (2019): Second-order variational equations for spatial point processes with a view to pair correlation function estimation. *CSGB Research Report* **2019-01**. Has appeared as *Spat. Stat.* (2019) **30**, 103-115.

8. Dvořák, J., Møller, J., Mrkvička, T. & Soubeyrand, S. (2019): Quick inference for log Gaussian Cox processes with non-stationary underlying random fields. *CSGB Research Report* **2019-03**. To appear in *Spat. Stat.*

9. Eriksen, R. & Kiderlen, M. (2019): Uniqueness of the measurement function in Crofton's formula. *CSGB Research Report* **2019-07**. *Adv. Appl. Math.*, in press.

10. Hessellund, K.B., Xu, G., Guan, Y. & Waagepetersen, R. (2019): Semi-parametric multinomial logistic regression for multivariate point pattern data. *CSGB Research Report* **2019-10**. Submitted.

11. Højgaard Jensen, M., Mallasto, A. & Sommer, S. (2019): Simulation of conditioned diffusions on the flat torus. *CSGB Research Report* **2019-04**. Has appeared in *Proceedings of the GSI 2019 Conference*, pp. 685-694.

12. Jensen, E.B.V. (2020): Stereological inference on particle shape from vertical sections. *CSGB Research Report* **2020-01**. Submitted.

13. Jensen, H.G., Lauze, F. & Darkner, S. (2019): Information-theoretic registration with explicit reorientation of diffusion-weighted images. *CSGB Research Report* **2019-05**. Submitted.

14. Sommer, S. & Bronstein, A. (2020): Horizontal flows and manifold stochastics in geometric deep learning. *CSGB Research Report* **2020-02**. Submitted.

15. Stehr, M. & Kiderlen, M. (2019): Asymptotic variance of Newton-Cotes quadratures based on randomized sampling points. *CSGB Research Report* **2019-02**. Submitted.

16. Stehr, M. & Rønn-Nielsen, A. (2019): Tail asymptotics of an infinitely divisible space-time model with convolution equivalent Lévy measure. *CSGB Research Report* **2019-09**. Submitted.

17. Vihrs, N., Møller, J. & Gelfand, A.E. (2020): Approximate Bayesian inference for a spatial point process model exhibiting regularity and random aggregation. *CSGB Research Report* **2020-05**.

SELECTED TALKS BY CSGB STAFF
1 January 2019 – 31 March 2020

Workshop on Diffusion MRI and Stochastic Geometry
Sandbjerg | 20 – 24 January 2019
Ute Hahn: Simulation based testing for stochastic geometry models

Workshop on Diffusion MRI and Stochastic Geometry
Sandbjerg | 20 – 24 January 2019
Markus Kiderlen: Fibre processes in stochastic geometry

15th International Congress for Stereology and Image Analysis
Aarhus | 27 – 30 May 2019
Tom Dela Haije: Imaging brain fibers with diffusion MRI

15th International Congress for Stereology and Image Analysis
Aarhus | 27 – 30 May 2019
Mads Stehr: Asymptotic variance of Newton-Cotes quadratures based on randomized sampling points

20th Workshop on Stochastic Geometry, Stereology and Image Analysis
Sandbjerg | 2 – 7 June 2019
Jesper Møller: The structure of stationary time series and point processes when constructing singular distribution functions

20th Workshop on Stochastic Geometry, Stereology and Image Analysis
Sandbjerg | 2 – 7 June 2019
Anne Marie Svane: Testing goodness of fit for point processes via topological data analysis

Stochastic Geometry Days
Avignon | 17 – 21 June 2019
Rasmus Waagepetersen: Multinomial logistic regression and regularized estimation for multivariate point processes

XI International Conference of Mathematical Physics in Armenia
Yerevan | 2 – 7 September 2019
Jesper Møller: Determinantal point processes and their usefulness in spatial statistics

Talk at Sino-Danish Center
Beijing | 3 September 2019
Jens R. Nyengaard: Glia cells

DALI 2019 – DATA, Learning and Inference
San Sebastian | 3 – 6 September 2019
Aasa Feragen: Uncertainties in diffusion MRI

DALI 2019 – DATA, Learning and Inference
San Sebastian | 3 – 6 September 2019
Aasa Feragen: Uncertainty quantification for manifold valued models

7th Annual Neuroscience & Neuroimaging Symposium
Beijing | 4 November 2019
Jens R. Nyengaard: Omics in neuroscience

CMStatistics 2019
London | 14 – 16 December 2019
Christophe Biscio: A general central limit theorem and a subsampling variance estimator for α-mixing point processes

CMStatistics 2019
London | 14 – 16 December 2019
Aasa Feragen: The geometry of graph space: Towards graph-valued statistics

CMStatistics 2019
London | 14 – 16 December 2019
Louis Gammelgaard Jensen: Doubly stochastic point processes in time and space as a model for photoactivated localization microscopy data

Quantitative Bioimaging Conference
Oxford | 6 – 9 January 2020
Rasmus Waagepetersen: Inferring correlations in inhomogeneous and multitype point patterns

Workshop on Manifold and Shape Stochastics
Sandbjerg | 23 – 27 February 2020
Jesper Møller: Statistics for point processes on the d-dimensional unit sphere

Workshop on Manifold and Shape Stochastics
Sandbjerg | 23 – 27 February 2020
Mathias Højgaard Jensen: Simulation of conditioned diffusions on Riemannian manifolds

APPENDIX

CSGB SCIENTIFIC STAFF

PROFESSORS

Eva B. Vedel Jensen (EBVJ)
Aasa Feragen (AF)
Jesper Møller (JM)
Mads Nielsen (MN)
Jens R. Nyengaard (JRN)
Jon Sporring (JS)
Rasmus P. Waagepetersen (RPW)


EBVJ


AF


JM


MN


JRN


JS


RPW

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Monika Golas (MG)
Ute Hahn (UH)
Markus Kiderlen (MK)
François Lauze (FL)
Andrew du Plessis (AP)
Jakob G. Rasmussen (JGR)
Ege Rubak (ER)
Anders Rønn-Nielsen (ARN)
Björn Sander (BS)
Stefan Sommer (SS)


JBA


SD


KADP


MG


UH


MK


FL


AP


JGR


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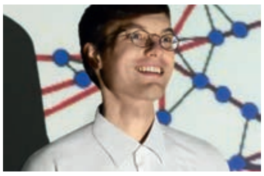
CH

AMS

ARAA

HSC

ADC

IDA

RKE

KBH

LGJ

MHJ

LK

NL

AJOM

RKN

FACP

MS

HJTS

HS

ASSISTANT PROFESSORS

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