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Second-order variational equations for spatial point processes with a view to pair correlation function estimation

Joint with Francisco Cuevas-Pacheco and Rasmus Waagepetersen

Moments of counts of objects for spatial point processes are typically expressed in terms of so-called joint intensity functions or Papangelou conditional intensity functions which are defined via the Campbell or Georgii-Nguyen-Zessin equations. In this talk, I will consider a third type of equation called variational equations introduced for parameter estimation in Markov random fields by [1]. A key feature of variational equations which have been proposed for spatial point processes compared to Campbell and Georgii-Nguyen-Zessin equations is that they are formulated in terms of the gradient of the log intensity or conditional intensity function rather than the (conditional) intensity itself.

Our first contribution is to establish second-order variational equations for second-order reweighted stationary (or istotropic) spatial point processes. Since the new variational equations are based on the gradient of the log pair correlation function, they take a particularly simple form for pair correlation functions of log-linear form. Our second contribution is to use this new equation to propose a new non-parametric estimator of the pair correlation. Standard estimators are the kernel density estimator or the recent orthogonal series estimator proposed by [2]. One of the drawback of these estimators is that they cannot be guaranteed to be non-negative. We propose to use our second-order variational equation to estimate coefficients in an orthogonal series expansion of the log pair correlation function. This ensures that the resulting pair correlation function estimator is non-negative. I will show on simulations and on real data how this new

estimator compares to existing ones.

References

[1] Almeida, M. P. and Gidas, B. (1993). A variational method for estimating the parameters of MRF from complete or incomplete data. *The Annals of Applied Probability*, 103-136.

[2] Jalilian, A., Guan, Y. and Waagepetersen, R. Orthogonal series estimation of the pair correlation function of a spatial point process, *to appear in Statistica Sinica*, 2019.