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Abstract

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# Monte-Carlo integration in any dimension using a single determinantal point process

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In a recent work [1], a particular projection determinantal point process (DPP), called Orthogonal Polynomial Ensemble (OPE), has been proposed to produce exactly $n$ quadrature points to estimate $\int_{[0,1]^d} f_d(u)\mu(\mathrm{d}u)$ where, for some $d \geqslant 1$, $f_d$ is a $d$-dimensional $\mu$-measurable function. The authors proved that their estimator satisfies a central limit theorem with explicit variance and rate of convergence $\sqrt{n^{1+1/d}}$ instead of the typical $\sqrt{n}$, under the main assumption that $f_d$ is continuously differentiable and compactly supported.

Using a product of Dirichlet type kernels instead of orthogonal polynomials, we extend this work when $\mu$ is the Lebesgue measure: to mimic a problem encountered in computer experiments, we investigate the problem of estimating $\mathscr{I}_\omega = \int_{[0,1]^\omega} f_\omega(u)\mathrm{d}u$ using a single configuration of $d$-dimensional points. We prove that our Dirichlet model is actually well-suited to such a problem, by exploiting the fact that any projection on a lower dimensional space of this model is distributed as a particular $\alpha$-DPP. For any $\omega = 1, \dots, d$, we eventually exhibit estimator of $\mathscr{I}_\omega$ which satisfies a central limit theorem with explicit variance and rate of convergence $\sqrt{n^{1+1/d}}$. Moreover, we relax the required assumptions for the function $f_\omega$: we only require that $f_\omega$ is "half"-differentiable (an assumption which is for instance satisfied for the $L^1$-norm) and do not impose that $f_\omega$ is compactly supported.

## References

[1] R. Bardenet and A. Hardy (2019) Monte Carlo with determinantal point processes. arXiv preprint arXiv:1605.00361, to appear in *Annal of Applied Probability*.