Abstracts SGSIA 2011

Semyon Alesker, Tel Aviv University

Theory of valuations and integral geometry

Theory of valuations is a classical part of convex geometry with strong relations to integral geometry. In recent years there was a considerable progress both in the theory of valuations itself, and its applications to integral geometry. In this talk we survey some of this progress. In the first part of the talk we describe some of the classical developments including the famous Hadwiger classification of isometry invariant continuous valuations on convex sets of a Euclidean space. In the second part of the talk we describe some of the more recent results: valuations on Hermitian spaces and the product structure on valuations. If time permits we explain their relation to integral geometry of Hermitian spaces.

Viktor Beneš, Charles University in Prague

Dimension reduction in marked fibre and surface processes

Joint work with Jakub Staněk and Ondřej Šedivý

We are interested in marked fibre and surface processes with multivariate marks and consider the dimension reduction problem. In Guan (2008), Guan and Wang (2010) general statistical theories on dimension reduction were generalized to spatial point processes driven by random fields. We continue these aims with marked fibre and surface processes instead of point processes and show that the situation may differ in some sense. Results of simulations are presented for the 3D Poisson-Voronoi tessellation and for a curve in a bounded planar domain being a solution of a SDE, both marked by a multivariate Gaussian random field.

References:

Y. Guan (2008) On consistent nonparametric intensity estimation for inhomogeneous spatial point processes. J. Amer. Statist. Assoc. 103, 483, 1238-1247.

Y. Guan, H. Wang (2010) Sufficient dimension reduction for spatial point processes directed by Gaussian random fields. J. R. Statist. Soc. B, 72, 3, 367-87.

Bartłomiej Błaszczyszyn, INRIA-ENS, Paris

Clustering, percolation and directionally convex ordering of point processes

Joint work with Dhandapani Yogeshwaran

In this talk, we want to examine the implications of directionally convex ordering of point processes on their percolative properties. We first relate directionally convex ordering of point processes to their clustering properties. Such a connection immediately yields heuristics on the afore-mentioned implications. Then, we give examples of sub-Poisson (super-Poisson) point processes - point processes that are smaller (larger) in directionally convex order than the Poisson point process. Using a weaker condition than directionally convex order, namely ordering of moment measures and void probabilities, we give a comparison of two critical radii for percolation of point processes. Further, we show that this weaker condition could be used to demonstrate non-trivial phase transition of percolation in weak sub-Poisson point processes.

Pierre Calka, Université de Rouen

Visibility estimates in the Euclidean and hyperbolic spaces

Joint work with Johan Tykesson

The classical continuum percolation model (or germ-grain model) is considered in the Euclidean space and hyperbolic plane. When grains are homogeneously distributed, the critical intensity for the existence of an infinite connected component of the occupied (resp. unoccupied) phase has been studied in the Euclidean case notably by R. Meester and R. Roy and this work has been recently extended to the hyperbolic case by J. Tykesson. In this talk, we are interested in the visibility percolation which consists in asking whether it is possible to see to infinity in at least one direction. In the Euclidean case, we show that it is almost surely not possible to see to infinity for any dimension unless the intensity measure of the underlying point process is modified accordingly. In the hyperbolic space, I. Benjamini, J. Jonasson, O. Schramm & J. Tykesson have proved that there exists a critical intensity for the visibility percolation. We present precise estimates for the distribution tail of the maximal visibility in the subcritical and critical regimes. Moreover, we consider some generalizations of the model and we show asymptotic results when the intensity goes to zero or to the critical value.

Jean-François Coeurjolly, Grenoble University

Revisiting the Takacs-Fiksel method for stationary marked Gibbs point processes

Joint work with Daniel Dereudre, Rémy Drouilhet and Frédéric Lavancier

Inference for parametric models and especially for parametric Gibbs models has known a large development during the last decade. The most popular method to estimate the parameters is certainly the maximum likelihood estimator (MLE). It involves an intractable normalizing constant, but recent developments in computational statistics, in particular perfect simulations, have made inference feasible for many Gibbs models (see [1]). Although the MLE suffers from a lack of theoretical justifications, some comparison studies, as in [4], have shown that it outperforms the other estimation methods. Nevertheless, the computation of the MLE remains very time-consuming and even extremely difficult to perform for some models. It is thus necessary to have quick alternative estimators at one's disposal, at least to propose relevant initial values for the MLE computation. The maximum pseudo-likelihood estimator (MPLE for short) constitutes one of them. Another estimation procedure is the Takacs-Fiksel estimator, which arose from e.g. [5]. It can be viewed, in some sense, as a generalization of the MPLE. As a matter of fact, the Takacs-Fiksel method is not very popular, nor really used in practice. The main reason is certainly its relatively poor performances, in terms of mean square error, observed for some particular cases as in [4]. However, we think that this procedure deserves some consideration for several reasons that we expose below.

The Takacs-Fiksel procedure is based on the Georgii-Nguyen-Zessin formula (GNZ formula for short). Empirical counterparts of the left and the right hand sides of this equation are considered, and the induced estimator is such that the difference of these two terms is close to zero. Since the GNZ formula is valid for any test functions, the Takacs-Fiksel procedure does not only lead to one particular estimator but to a family of estimators, depending on the choice of the test functions. This flexibility is the main advantage of the procedure. We will present several examples in this talk. In particular, first, this procedure can allow us to achieve estimations that likelihood-type methods cannot. As an example, we focus on the quermass model, which

gathers the area interaction point process as a particular case (see [8]). This model is some-times used for geometric random objects. From a data set, one typically does not observe the point pattern but only some geometric sets arising from these points. The non-observability of the points makes the likelihood-type inference unfeasible. As we will show, this problem may be solved thanks to the Takacs-Fiksel procedure, provided that the test functions are chosen properly.

Another motivation is the possibility to choose test functions depending on the Hamiltonian in order to construct quicker estimators which do not require the computation of an integral for each value of the parameter. This improvement appears crucial for rigid models, such as those involved in stochastic geometry (see [3]), for which the MLE is prohibitively time-consuming and the MPLE still remains difficult to implement. Moreover, for some models, it is even possible to obtain explicit estimators which do not require any simulation nor optimization. We illustrate this for the Strauss and multi-Strauss models and for estimation of the "singleton" parameter estimation.

Some asymptotic properties of the Takacs-Fiksel procedure have already been investigated in two previous studies: one by L. Heinrich in [6] and the one by J.-M. Billiot in [2]. These pa-pers have different frameworks and are based on different tools, but they both involve regularity and integrability type assumptions on the Hamiltonian and a theoretical condition which ensures that the contrast function (associated to the Takacs-Fiksel procedure) has a unique minimum. In [6], the consistency and the asymptotic normality are obtained for a quite large class of test functions. These results are, however, proved under the Dobrushin condition (see Theorems 2 and 3 in [6]) which implies the uniqueness of the underlying Gibbs measure and some mixing properties. This condition imposes a reduction of the space of possible values for the parame-ters of the model. In [2], the author focuses only on pairwise interaction point processes (which excludes the quermass model for example). The author mainly obtained the consistency for a specific class of test functions. In the case of a multi-Strauss pairwise interaction point process, the author also proved that the identifiability condition holds for the class of test functions he considered.

In contrast, our asymptotic results are proved in a very general setting, i.e. for a large class of stationary marked Gibbs models and test functions. The method employed to prove asymptotic normality is based on a conditional centering assumption generalized to certain spatial point processes in [7]. The main restriction that this method induces is only the finite range of the Hamiltonian. There are no limitations on the space of parameters and, in particular, the possible presence of phase transition does not affect the asymptotic behavior of the estimator. Moreover, the test functions may depend on the parameters. The general assumptions involved in the asymptotic results will be discussed and a short simulation study will be presented.

References:

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Richard Cowan, University of Sydney

Topological relationships in spatial tessellations

Joint work with Viola Weiss

Tessellations of $\mathbb{R}^3$ that use convex polyhedral cells to fill the space can be extremely complicated. This is especially so for tessellations which are not 'facet-to-facet', that is, for those where the facets of a cell do not necessarily coincide with the facets of that cell's neighbours. Adjacency concepts between neighbouring cells (or between neighbouring cell elements) are not easily formulated when facets do not coincide. In this talk, we briefly summarise our systematic study of these topological relationships when a tessellation of $\mathbb{R}^3$ is not facet-to-facet.

Daryl J. Daley, Australian National University, Canberra

Isotropy and second order properties: mathematical observations

Joint work with Emilio Porcu

Basically, one can best regard isotropy as being a componentwise concept, i.e. a partition of the total dimension d (e.g. 3 or 4 for space-time) into subspaces (e.g. 1 dimension for time, and 2 or 3 for space), but space itself may be isotropic in the plane but not the vertical direction. The main interest in isotropy is the reduction it may afford in specification because of 'homogeneity' in whatever is the number of dimensions. In this number of dimensions, one may also afford 'dimension random walk' as the numerical analysts are interested, and for second-order properties there are then curious phenomena best described via what we call the Schoenberg measure in the univariate reduction of the spectral measure of Bochner's representation for positive definite functions. Matheron exploited ideas of dimension change through what he called Montee and Descente: these do not necessarily apply even to completely monotone functions.

Jiří Dvořák, Charles University in Prague

Comparison of moment estimation methods for stationary spatial Cox processes

In the poster we consider moment estimation methods for stationary spatial Cox point processes. These represent a simulation-free faster-to-compute alternative to the computationally intense

maximum likelihood estimation. First we give an overview of the available methods i.e. the minimum contrast method and the composite likelihood and the Palm likelihood approaches. Further we present results of a simulation study in which performance of these estimating methods was compared for planar point processes. In the simulation study we considered two types of point processes - Thomas process and log-Gaussian Cox process - to cover different types of clustering and inter-point interactions.

Daniel Gentner, Karlsruhe Institute of Technology

Palm theory, mass transports and ergodic theory for group stationary processes

This poster is about random measures stationary (distributionally invariant) with respect to a possibly non-transitive and possibly non-unimodular group action. We shall present some recent results in this area. E.g. we show how typical objects of spatial processes may be extracted by using a new object in Palm theory - the cumulative Palm measure. To highlight the relevance of this object, we provide interesting links to a general mass-transport principle and to new ergodic theorems for such random measures. The theory allows e.g. the inspection of isometry-stationary random partitions on non-compact Riemannian manifolds as we shall illustrate.

Mohammad Ghorbani, Aalborg University

Second-order analysis of structured inhomogeneous spatio-temporal point processes

Joint work with Jesper Møller

Statistical methodology for spatio-temporal point processes is in its infancy. We consider second-order analysis based on pair correlation functions and K-functions for first general inhomogeneous spatio-temporal point processes and second inhomogeneous spatio-temporal Cox processes. Assuming spatio-temporal separability of the intensity function, we clarify different meanings of second-order spatio-temporal separability. One is second-order spatio-temporal independence and relates e.g. to log-Gaussian Cox processes with an additive covariance structure of the underlying spatio-temporal Gaussian process. Another concerns shot-noise Cox processes with a separable spatio-temporal covariance density. We propose diagnostic procedures for checking hypotheses of second-order spatio-temporal separability, which we apply on simulated and real data (the UK 2001 epidemic foot and mouth disease data).

Yongtao Guan, Yale School of Public Health

On consistent nonparametric intensity estimation for inhomogeneous spatial point processes

A common nonparametric approach to estimate the intensity function of an inhomogeneous spatial point process is through kernel smoothing. When conducting the smoothing, one typically uses events only in a local set around the point of interest. The resulting estimator, however, is often inconsistent since the number of events in a fixed set is of order one for spatial point processes. In this paper, we propose a new covariate-based kernel smoothing method to estimate the intensity function. Our method defines the distance between any two points as the difference between their associated covariate values. Consequently, we determine the kernel weight for a given event of the process as a function of its new distance to the point of interest. Under some suitable conditions on the covariates and the spatial point process, we prove that our new estimator is consistent for the true intensity. To handle the situation with high-dimensional

covariates, we also extend sliced inverse regression, which is a useful dimension-reduction tool in standard regression analysis, to spatial point processes. Simulations and an application to a real data example are used to demonstrate the usefulness of the proposed method.

Lothar Heinrich, Augsburg University

On the Brillinger-mixing property of stationary point processes

We consider a stationary infinite-order point process $\Psi \sim P$ on $\mathbb{R}^d$ satisfying the additional assumption that, for each $k \geq 2$, the reduced kth-order factorial cumulant measure $\gamma_{red}^{(k)}(\cdot)$ has finite total variation $\|\gamma_{red}^{(k)}\|$ on $(\mathbb{R}^d)^{k-1}$. This property of Ψ, which is attributed to D. R. Brillinger, expresses weak mutual correlations between the numbers taking the counting measure Ψ in distant sets. This condition is essential to prove asymptotic normality of shot noise processes, moment estimators, empirical product densities etc., see references in [2]. The aim of the talk is to compare *Brillinger-mixing* with other mixing conditions. Correcting a result and its proof in [1], we first prove that Brillinger-mixing implies the usual *mixing* of $\Psi \sim P$ provided that P is uniquely determined by its one-dimensional moment sequences. If, in addition, $\|\gamma_{red}^{(k)}\| \leq c^k k!$ for some $c > 0$ and any $k \geq 2$, we show that the tail-σ-algebra of Ψ is trivial, i.e. Ψ is *regular* or has *short-range correlations* as statistical physicists say. On the other hand, we formulate a condition in terms of the coefficient of *absolute regularity* which implies $\|\gamma_{red}^{(k)}\| < \infty$ for a fixed $k \geq 2$. Finally, we give a continuum of distinct Neyman-Scott cluster processes having all the same factorial moment measures as a simple example of an indeterminate moment problem for point processes. Detailed proofs are given in [2].

References:

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Kateřina Staňková Helisová, Czech Technical University in Prague

Estimating parameters in Quermass-interaction process

Joint work with David Dereudre and Frédéric Lavancier

Consider a random set observed in a bounded window $W \subset \mathbf{R}^2$. The set is given by a union of interacting discs. It is described by the Quermass-interaction process, i.e. the density of any finite configuration $\mathbf{x} = (x_1, ..., x_n)$ of the discs $x_1, ..., x_n$ with respect to the probability measure of a stationary random-disc Boolean model with the intensity of the centers z is given by

$$f_\theta(\mathbf{x}) = \frac{z^n \exp\{\theta_1 A(U_\mathbf{x}) + \theta_2 L(U_\mathbf{x}) + \theta_3 \chi(U_\mathbf{x})\}}{c_\theta},$$

where $A(U_\mathbf{x})$ denotes the area, $L(U_\mathbf{x})$ the perimeter and $\chi(U_\mathbf{x})$ the Euler-Poincare characteristic of the union $U_\mathbf{x}$ composed of the discs from the configuration $\mathbf{x}$. Further, $\theta = (\theta_1, \theta_2, \theta_3)$ is a vector of parameters and c_θ is a normalizing constant.

In this contribution, we describe a method for estimating the parameters $\theta_1, \theta_2, \theta_3$ and z studied in [1] based on Takacs-Fiksel procedure, and compare it with MCMC maximum likelihood method described in [2].

References:

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Christian Hirsch, Ulm University

Connectivity of random geometric graphs related to minimal spanning forests

Joint work with David Neuhäuser and Volker Schmidt

For any locally finite set $M \subset \mathbb{R}^d$, $d \geq 2$, the minimal spanning forest $\mathrm{MSF}(M)$ is a geometric graph with vertex set M whose edge set is constructed as follows. Any two vertices $x, y \in M$ are connected if and only if there does not exist an integer $m \geq 2$ and a sequence $x_0 = x, \ldots, x_m = y \in M$ such that

$$|x_k - x_{k+1}| < |x - y| \qquad \text{for all } k = 0, \ldots, m - 1 \,. \tag{1}$$

Aldous and Steele [1] conjectured that $\mathrm{MSF}(X)$ is almost surely connected if $X \subset \mathbb{R}^d$ is a homogeneous Poisson point process. This conjecture was proven in [2] for dimension $d = 2$. However, it remains open for $d \geq 3$ (and for non–Poisson point processes X).

In this talk we investigate the connectivity of random geometric graphs which can be seen as approximations of minimal spanning forests. For any $n \geq 2$ and $M \subset \mathbb{R}^d$ locally finite, we consider the graph $G_n(M) \supset \mathrm{MSF}(M)$ with vertex set M, where any two vertices $x, y \in M$ are connected if and only if there does not exist an integer $m \in \{2, \ldots, n\}$ and a sequence $x_0 = x, \ldots, x_m = y \in M$ such that (1) holds. Note that

$$G_2(M) \supset G_3(M) \supset \ldots \supset \cap_{n=2}^\infty G_n(M) = \mathrm{MSF}(M) \,.$$

Furthermore, $G_2(M)$ is the so–called β–skeleton induced by M for $\beta = 2$.

We derive sufficient criteria for the connectivity of $G_n(X)$, which are satisfied for a large class of point processes $X \subset \mathbb{R}^d$. These criteria are related to a descending–chains condition (see [4]) and to some annulus-type continuum percolation model considered e.g. in [3]. In this way, connectivity of $G_n(X)$ can be shown not only for the point processes discussed in [4]. But, using results presented in [5], we can prove that $G_n(X)$ is almost surely connected for any stationary point processes X with finite range of dependence and absolutely continuous second factorial moment measure.

References:

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Stephan Huckemann, Georgia Augusta University Goettingen

Ziezold means for shape spaces

A large number of concepts for mean direction, mean shape, etc. have been proposed in the literature. It turns out that they are obtained from three fundamental types, one of which introduced by H. Ziezold (1994) has found little interest, thus far. We will see that this mean performs very well in view of geometry, computation and statistics. In particular, it allows for a Central Limit Theorem for mean geodesics in a shape space yielding in application to forest biometry, discrimination of poplar genotypes by observations of brief growth episodes only.

Daniel Hug, Karlsruhe Institute of Technology

Random mosaics in high dimensions

Joint work with Julia Hörrmann

Previously, we considered asymptotic distributional results for random tessellations where some bound for a geometric functional (like the volume) goes to infinity. These results concern random polytopes which arise as particular cells of random tessellations and are related to isoperimetric and stability problems in geometry. (joint work with Rolf Schneider)

There are now also some new investigations of distributional properties of the volume (say) of certain random cells when the dimension of the space is increasing. In this asymptotic framework, Poisson Voronoi and hyperplane tessellations exhibit a surprisingly different behaviour.

Abdollah Jalilian, Shahid Beheshti University, Teheran

Model checking for Neyman-Scott Processes using Laplace functionals

Joint work with Mohammad Q. Vahidi-Asl

Due to computational difficulties the implementation of the standard martingale residuals for Neyman-Scott processes is impractical. Since the Laplace functional of a Neyman-Scott process can be expressed in a closed form, one can use the Laplace functional to assess the goodness-of-fit and revealing inadequacy of fitted Neyman-Scott models. In the present work, we developed a residual analysis for inhomogeneous Neyman-Scott models based on Laplace functionals. A simulation study and real data examples demonstrate the performance of the method.

Kristjana Ýr Jónsdóttir, Aarhus University

Lévy based modelling in brain imaging

Joint work with Anders Rønn-Nielsen, Kim Mouridsen and Eva B. Vedel Jensen

Traditional methods of analysis in brain imaging based on Gaussian random field theory may leave small, but significant changes in the signal level undetected, because the assumption of Gaussianity is not fulfilled. In group comparisons, the number of subjects in each group is usually small so the alternative strategy of using a non-parametric test may not be appropriate either because of low power. We propose to use a flexible, yet tractable model for a random field, based on kernel smoothing of a so-called Lévy basis. The resulting field may be Gaussian but there are many other possibilities, e.g. random fields based on Gamma, inverse Gaussian and

normal inverse Gaussian (NIG) Lévy bases. We show that it is easy to estimate the parameters of the model and accordingly to assess by simulation the quantiles of a test statistic. A finding of independent interest is the explicit form of the kernel function that induces a covariance function belonging to the Matérn family.

Jürgen Kampf, Technische Universität Kaiserslautern

On weighted parallel volumes and spaces of continuous functionals of convex bodies

Weighted parallel volumes are a class of functionals that generalize the Wills functional, see [2]. They map a convex body K to $\int_{[0,\infty)} V_d(K + \lambda B) \, d\rho(\lambda)$, where ρ is a signed measure on $[0,\infty)$ and B is a fixed convex body, V_d denotes the Lebesgue measure and $\mathcal{K}$ denotes the set of convex bodies. We will show that the measure ρ is determined uniquely by the weighted parallel volume induced by it.

Alesker [1] proved that the functionals which map a convex body K to the real number $V_d(K+A)$, where A is a convex body, spann a dense subspace in the space of all continuous, translation-invariant valuations from the set of all convex bodies to the real line. Using the result mentioned above we can give a suprisingly simple proof for the fact that we can replace "valuations" by "functionals" in Alesker's result, if we allow A to be a non-convex body.

References:

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Wolfgang Karcher, Ulm University

A central limit theorem for the volume of the excursion sets of associated stochastically continuous stationary random fields

In [1], the multivariate central limit theorems (CLTs) for the volumes of excursion sets of stationary quasi-associated random fields on $\mathbb{R}^d$ is proved. We extend their results by considering associated stochastically continuous stationary random fields which need not necessarily have second moments. For certain max-stable and sum-stable random fields with spectral representation, we provide simple conditions on the tail behavior of the kernel functions such that the assumptions of the CLT are satisfied.

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Kiên Kiêu, INRA, Jouy-en-Josas

Modelling a 3D active genome as a Gibbs marked point process

Joint work with Clémence Kress

We will present a new model accounting for the spatial organization and the transcription of the genome in the mammalian cell nucleus. In this model, genome regions are represented as sequences of points with marks reflecting their transcriptional activity. The probabilistic distribution of the marked points is written as a Gibbs distribution with several potentials representing different types of interactions between nuclear elements.

The considered interactions are: volume exclusion (limiting the local chromatin density), chromatin elasticity (limiting the distance between consecutive points lying on the same chromosome), binding of chromatin to the nuclear envelope, repression of transcription due to local crowding and clustering of highly transcribed genomic regions. In addition to encompass a wide range of interactions compared to previous 3D DNA models, our model takes into account the heterogeneity of the chromatin fibers concerning elasticity, transcription and interactions with the nuclear envelope. More precisely, some interaction parameters vary along the sequence of points. Hence the proposed model is of very high dimension with a number of parameters of the same order as the number of points.

An algorithm for simulating the model together with some simulation results will be presented. Finally some issues relative to fitting from genome-wide contact data will be discussed.

Petra Kochová, University of West Bohemia, Pilsen

Determination of anisotropy of three-dimensionally oriented line systems

Joint work with Robert Cimrman, Kirsti Witter and Zbyněk Tonar

Two- as well as three-dimensional analyses of spatial structures, their orientation and preferential directions of fibre systems are very common tasks in various fields, such as the paper and textile industry, medicine, biology and tissue modelling. The amount, arrangement and orientation of blood vessels affects the nutritional support of biological tissues. Similarly, the topology of collagen and elastin fibres or cellulose fibres determines the mechanical properties of animal or plant tissue matrix. Quantitative description of the degree of an/isotropy of fibres and other linear structures helps us to understand their growth and function. In our study, we present a novel χ^2-based method for evaluating the anisotropy of line systems that involves comparing the observed length densities of lines with the discrete uniform distribution of an isotropic line system using the χ^2-test. This method, programmed as an open source software, was used to determine the rose of directions, preferential directions and the level of anisotropy of linear systems representing the blood microvessels in samples of human brains (cortex, subcortical grey matter and white matter). The results showed that the microvascular bed in the cortex was closer to an isotropic network, while the microvessels supplying the white matter appeared to be an anisotropic and direction-sensitive system. The advantage of our χ^2 method is its high correlation with the number of preferential directions of the line system.

Guenter Last, Karlsruhe Institue of Technology

Chaos expansion of geometric Poisson functionals

Joint work with Mathew Penrose

The space of square integrable functions of a general Poisson process is isomorphic to a suitable Fock space. This isomorphism can be expressed in terms of expected iterated difference operators. It can be used to derive covariance identities and explicit chaos expansions. The first part of the talk will be devoted to the basic theory. The second part deals with specific examples.

Frédéric Lavancier, Université de Nantes

Inference for union of interacting discs by the Takacs-Fiksel method

Let us consider a set of discs with random location and random radius. We assume to observe only the random set formed by the union of these discs. In particular, the discs themselves, their center, or their radius are not observable. When the centers are distributed as a Poisson point process, this model reduces to the well-known Boolean model. More generally, the centers are assumed to be distributed as a Gibbs point process, where the Hamiltonian depends on some geometrical features of the random set (like the perimeter, the area, the number of connected components,...). These models allow to construct a richer class of random sets than the Boolean model. They include the well-known area-process and the Quermass process.

Inference for these models is difficult because the points are not observable. In particular, it is impossible to estimate all parameters by maximum likelihood. We present a new procedure, based on the Takacs-Fiksel method, which allows to estimate all parameters. Some theoretical aspects will be presented and the validity of the procedure will be confirmed by a simulation study.

André Liebscher, Technische Universität Kaiserslautern

Characterization of the local strut thickness of open foams based on 3D image data

Open celled foam structures are predesignated for the use as catalytic support in automotive industry or as filters in steel casting technology. The first step to understand the properties of such materials is the investigation of their microstructure. Owing to their high porosity, X-ray computed tomography (CT) is especially suited for this purpose.

An open celled foam is comprised of polyhedral cells, whose edges form an interconnected network. Techniques to separate individual cells from each other and to measure mean values of surface area, diameter, etc. for the foams cells, windows and struts are well established. The questions that will be tackled in this talk is: How to quantify the local variation of the strut thickness?

To measure the local strut thickness we introduce a skeletonization based topological decomposition of the foam structure into its vertices and struts. This allows for the estimation of the length and the local thickness of the individual strut segments. Joining both estimates yields a thickness profile for each strut. Conflating those profiles based on the struts' length classes results in a strut profile for the entire foam. We develop a model for the foams' strut profile which is an essential corner stone for more realistic foam models.

Marie-Colette van Lieshout, CWI Amsterdam

Simulation algorithms for polygonal Markov fields applied to image segmentation

We introduce a class of Gibbs-Markov random fields built on regular tessellations that can be understood as discrete counterparts of Arak-Surgailis polygonal fields.We focus first on consistent polygonal fields, for which we show consistency, Markovianity, and solvability by means of dynamic representations. Next, we develop simulation dynamics for their general Gibbsian modifications, which cover most lattice-based Gibbs-Markov random fields subject to certain mild conditions. Applications to foreground-background image segmentation problems are discussed.

Kimberly McGarrity, Delft University of Technology

Quantification and modeling of microstructural banding in steel from serial sectioned optical microscopy

Joint work with Jilt Sietsma and Geurt Jongbloed

The mechanical properties of steel are directly related to the 3D microstructure. However, the connection between the 3D modeling and experimental data is tenuous. Experimental data is mostly 2D in the form of optical microscopy, TEM, SEM, EBSD, etc. This 2D data cannot accurately represent what is happening inside the microstructure. The aim of our work is to create a stochastic model that requires a single 2D input image and will extract the relevant 3D parameters that reliably represent the mechanical properties. To achieve this aim, we have serial sectioned data from two different microstructurally banded materials in each of the three directions: rolling, transverse and normal. We also have several 2D and 3D banded microstructures created by Voronoi tessellations. We have quantified the degree of banding in the 2D images using four parameters that are bounded on zero and one. These parameters are extendible to 3D and provide the first links between the 2D and 3D data. We are currently focused on describing the bands in the microstructure with shape parameters and stereological descriptors. From here we will create a stochastic model to represent the observed banded microstructures and incorporate the mechanical properties due to the banding of the microstructures into the model. Finally, the model will be extended to receive any general 2D microstructure and, by means of statistical analysis, link the 3D characteristics to the mechanical properties.

Robert M. Mnatsakanov, West Virginia University

Image reconstruction from its Radon transform

Joint work with Shengqiao Li

In this talk the problem of recovering a multivariate function (image) as well as the shape of the region on the plain from its Radon transform (projections) will be discussed. It is known that the tomographic measurements of a body can be converted into its moments (see Goncharov (1988) and Milanfar *et al.* (1996)) and this fact is fundamental in seismic, medical, and magnetic resonance imaging. In other words, the x-rays of an object can be used to specify the moments of the underlying object. We suggest to use the moments specified in this way and recover the shape of the object being imaged. In particular, we applied the moment-recovered construction from Mnatsakanov (2011) to invert the Radon transform. The performance of our approximation

is demonstrated via the simulation study.

References:

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Ilya Molchanov, Universität Bern

Regularity conditions in the realisability problem in applications to point processes and random closed sets

Joint work with Raphael Lachieze-Rey

The talk addresses the existence issue for a rather general random element whose distribution is only partially specified. The technique relies on the existence of a positive extension for linear functionals accompanied by additional conditions that ensure the regularity of the extension needed for interpreting it as a probability measure. It is shown in which case the extension can be chosen to possess some invariance properties.

The results are applied to obtain existence results for point processes with given correlation measure and random closed sets with given two-point covering function or contact distribution function. The regularity conditions ensure that the obtained point processes are indeed locally finite and random sets have closed realisations.

Jesper Møller, Aalborg University

Transforming spatial point processes into Poisson processes using random superposition

Most finite spatial point process models specified by a density are locally stable, implying that the Papangelou intensity is bounded by some integrable function β defined on the space for the points of the process. It is possible to superpose a locally stable spatial point process X with a complementary spatial point process Y to obtain a Poisson process $X \cup Y$ with intensity function β. Underlying this is a bivariate spatial birth-death process (X_t, Y_t) which converges towards the distribution of (X, Y). We study the joint distribution of X and Y, and their marginal and conditional distributions. In particular, we introduce a fast and easy simulation procedure for Y conditional on X. This may be used for model checking: given a model for the Papangelou intensity of the original spatial point process, this model is used to generate the complementary process, and the resulting superposition is a Poisson process with intensity function β if and only if the true Papangelou intensity is used. Whether the superposition is actually such a Poisson process can easily be examined using well known results and fast simulation procedures for Poisson processes. We illustrate this approach to model checking in the case of a Strauss process

Tomáš Mrkvička, University of South Bohemia, České Budějovice

Two step estimation for Neyman-Scott point process with inhomogeneous cluster centers

This talk is concerned with parameter estimation for Neyman-Scott point process with inhomogeneous cluster centers. The inhomogeneity depends on spatial covariates. The regression parameters are estimated in the first step using Poisson likelihood score function. Three estimation procedures (minimum contrast method based on a modified K function, composite likelihood method, Bayesian method) are introduced for estimation of clustering parameters in the second step. The performance of the estimation methods are studied and compared via simulation study. The work is motivated and illustrated by ecological studies of fish spatial distribution in the inland reservoir.

Werner Nagel, Friedrich Schiller University Jena

Markovian processes of tessellations that are generated by cell division

We consider processes of random tessellations of the Euclidean space where the cells are convex polytopes. These cells can be divided individually at random times by random hyperplanes such that the divided cell gives birth to two new cells. Thus we obtain a pure jump process (on the time axis) where the states are tessellations.

This model is rather flexible since the distributions of the life-times of the cells (closely related to Cowan's selection rule) and the distribution of the dividing hyperplane (Cowan's division rule) can be chosen in different ways. But on the other hand, most of these models seem to resist a theoretical treatment. At the moment, the STIT tessellation model (STable under the operation of ITeration of tessellations) appears to be the most fruitful one and potentially a reference model for applications.

After a general introduction the lecture will focus on several results for STIT tessellation processes such as stationarity and ergodicity in space and in time.

Michael Nolde, Augsburg University

Asymptotic variance relations in stationary normal tessellations of the plane

A planar random tessellation T induces three planar point processes, namely the pp N_0^T of the nodes, the pp N_1^T of the edge-midpoints, and the pp N_2^T of the cell-centroids of T. In stationary normal tessellations - where each vertex is shared by three cells, and each edge is common to two cells - their intensities satisfy (cf.[1]) the simple relationships: $2\lambda_2 = \lambda_0$ and $3\lambda_2 = \lambda_1$.

As we could show in [2], there are similar exact equalities for the asymptotic variances: $4\sigma_2^2 = \sigma_0^2$ and $9\sigma_2^2 = \sigma_1^2$, provided that one of the asymptotic variances (and therefor all of them) exists. Thereby the existence can be shown for α-mixing tessellations, in which the number of edges, that hit a unit square, has a finite moment of order higher than two. In this case we can prove, that the joint distribution of N_0^T, N_1^T, and N_2^T on a growing window is asymptotically normal.

References:

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Zbyněk Pawlas, Charles University in Prague

Inference for geostatistically marked point processes

Let $\Phi = \{X_i\}$ be a stationary point process on d-dimensional Euclidean space $\mathbb{R}^d$ and let $\{Z(x) : x \in \mathbb{R}^d\}$ be a d-dimensional real-valued stationary random field, which is independent of Φ. We construct a marked point process Ψ from Φ and $\{Z(x)\}$. It is obtained by so called geostatistical (or external) marking: $\Psi = \{(X_i, Z(X_i))\}$. This model is also known as random field model. It was probably first used by A. F. Karr in [1]. We assume that a single realization of Ψ is observed in a convex compact sampling window W. It means that the measurements $Z(X_i)$ at random points $X_i \in \Phi \cap W$ are available. Our aim is to draw inference about the random field $\{Z(x)\}$ on the basis of this observation. We investigate the estimation of the mean $\mu = \mathbb{E}Z(o)$, covariance function $R(h) = \mathrm{Cov}(Z(o), Z(h))$ and marginal distribution function $F(t) = \mathbb{P}(Z(o) \leq t)$. The estimation of μ and R was treated in [1] under the assumption that Φ is a Poisson process. The estimation of F was studied in [2] under strong mixing conditions on the model. We present asymptotic results under different large sample limiting regimes (infill asymptotics and increasing domain asymptotics). Another problem is to reconstruct unobserved values of $\{Z(x)\}$. This state estimation problem was considered in [1] for Poisson samples. We discuss possible generalizations for non-Poisson point processes.

References:

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[2] Pawlas, Z. (2009): Empirical distributions in marked point processes, *Stochastic Process. Appl.* **119**, 4194–4209.

Mathew Penrose, University of Bath

Limit theorems in stochastic geometry with applications

For an empirical point process governed by a probability density function in d-space, consider functionals obtained by summing over each point some function which is locally determined. General laws of large numbers and central limit theorems for such functionals are known. We discuss such results, their extensions to point processes in manifolds, associated local limit theorems, and applications to particular functionals such as multidimensional spacings statistics, dimension estimators and entropy estimators.

Jeffrey Picka, University of New Brunswick

Random trajectory models for disordered physical systems

Forest fires and powder flows are examples of physical phenomena that can be represented by time series of disordered spatial patterns. Scientists generally represent these fires and flows by means of deterministic models which cannot be scientifically validated. By introducing stochastic elements into these deterministic models, it is possible to establish a scientific procedure for validating these models by means of spatial statistical inference.

Michaela Prokešová, Charles University in Prague

On parameter estimation for spatial Cox process models

In the talk we will discuss parameter estimation for homogeneous and inhomogeneous spatial Cox processes. We first give an overview of the moment estimation methods for the homogeneous case and then we discuss parameter estimation for Cox models with inhomogeneity introduced by location dependent thinning. In the first step the inhomogeneity is estimated by means of the Poisson likelihood. Then conditionally on the estimated inhomogeneity parameters the interaction (clustering) parameters are estimated. This can be done by the minimum contrast method or by different versions of the composite or Palm likelihood methods generalized from the homogeneous case. We discuss the properties of the resulting estimators and compare their efficiency by a simulation study.

Jakob Gulddahl Rasmussen, Aalborg University

A sequential point process model for spatial point patterns with linear structures

Many observed spatial point patterns contain points placed roughly on line segments. Point patterns exhibiting such structures can be found for example in archaeology (locations of bronze age graves in Denmark) and geography (locations of mountain tops). We consider a particular class of point processes whose realizations contain such linear structures. This point process is constructed sequentially by placing one point at a time. The points are placed in such a way that new points are often placed close to previously placed points, and the points form roughly line shaped structures. We consider Markov chain Monte Carlo based estimation for this class of point processes in a Bayesian setup. This is exemplified by real data.

Jan Rataj, Charles University in Prague

On generalised normal measures and their estimation

Given a sufficiently regular (i.e., polyconvex) compact set in a Euclidean space, its normal measure is the measure on the unit sphere obtained as image under the Gauss map (defined almost everywhere) of the surface area measure on the boundary. This notion has a natural generalisation for sets with finite perimeter. We present some formulas useful for estimation of certain functionals of these generalised area measures, using the covariogram and its extensions.

Claudia Redenbach, Technische Universität Kaiserslautern

Random tessellations - open problems from an applied point of view

Random tessellations form a versatile class of models in stochastic geometry. Well-established models include Voronoi and Laguerre tessellations, hyperplane tessellations, and STIT tessellations. They are applied to such diverse fields as the modelling of cellular materials, biological cells, road systems or animal territories.

In practice, tessellation models are fit to the observed data using geometric characteristics which can be determined experimentally or by means of image analysis. These characteristics are then compared to the characteristics of a tessellation model under consideration. Mean values and even distributions of some geometric characteristics of random tessellations can be obtained analytically. However, several questions whose answers are of particular interest from a user's point of view are still open.

In this talk, we will present some of these open problems which are motivated by the application of random tessellations as models for cellular materials. These include problems related to the choice of distance measures between random tessellations. Furthermore, we investigate the parallel sets of the edges or facets of a tessellation which can be used as models for open or closed foams, respectively. We will discuss the state of the art and name several problems whose solution would significantly reduce the complexity of model fitting procedures.

Ege Rubak, Aalborg University

Statistical aspects of determinantal point processes

Joint work with Jesper Møller and Frédéric Lavancier

Determinantal point processes are largely unexplored in statistics, though they possess a number of appealing properties and have been studied in mathematical physics, combinatorics, and random matrix theory as described in the excellent survey by Hough et al. (2009)

We consider statistical aspects of determinantal point processes defined on $\mathbb{R}^d$, with a focus on $d = 2$. Determinantal point processes are defined by a function C satisfying certain regularity conditions and they possess a number of appealing properties:

(a) All orders of moments of a determinantal point process are described by certain determinants of matrices with entries given in terms of C.

(b) A one-to-one smooth transformation or an independent thinning of a determinantal point processes is also a determinantal point processes.

(c) A determinantal process can easily be simulated, since it is a mixture of 'determinantal projection processes'.

(d) A determinantal point process restricted to a compact set has a density (with respect to a Poisson process) which can be expressed in closed form including the normalizing constant.

Moreover, determinantal point processes can be viewed as Gibbs point processes and they are flexible models for repulsive interaction. For a general Gibbs point process, the moments are not expressible in closed form, the density involves an intractable normalizing constant, and rather time consuming Markov chain Monte Carlo methods are needed for simulations and approximate likelihood inference.

In this work we describe how to simulate determinantal point processes in practice and investigate how to construct parametric models. Furthermore, different inferential approaches based on both moments and the likelihood are studied.

References:

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Håvard Rue, Norwegian University of Science and Technology, Trondheim

Spatial modeling and computation using SPDE:s

In this talk I will review our ongoing work using SPDE:s in spatial modelling and as a tool to con-

struct computational efficient spatial (Gaussian) models. In short, the SPDE approach provides an explicit link between (some) Gaussian fields and Gaussian Markov random fields, meaning that the modelling can be done using Gaussian fields, but the computations can be done using the corresponding Gaussian Markov random field which allow for sparse matrix algebra. Extensions to non-stationary models, spatio-temporal models, fields on manifolds, are direct extensions of this idea.

Gennady Samorodnitsky, Cornell University

Large deviations for Minkowski sums of heavy-tailed random compact sets

Joint work with Thomas Mikosch and Zbyněk Pawlas

We prove large deviation results for Minkowski sums S_n of iid random compact sets, both convex and non-convex, where we assume that the summands have a regularly varying distribution and either finite or infinite expectation. The results confirm the heavy-tailed large deviation heuristics: "large" values of the sum are essentially due to the "largest" summand.

Volker Schmidt, Ulm University

On the distribution of typical shortest–path lengths in connected random geometric graphs

Joint work with Daniel Neuhäuser, Christian Hirsch and Catherine Gloaguen

We consider random geometric graphs in $\mathbb{R}^2$ represented by their random edge set G which is assumed to be connected and stationary. Furthermore, we consider two stationary Coxian point processes X_H and X_L in $\mathbb{R}^2$ whose random intensity measures are concentrated on G. We assume that (i) X_H and X_L are conditionally independent given G and (ii) their random intensity measures are proportional to the one–dimensional Hausdorff measure ν_1 on G, i.e., $\mathbb{E}X_H(B) = \lambda_\ell \mathbb{E}\nu_1(B \cap G)$ and $\mathbb{E}X_L(B) = \lambda'_\ell \mathbb{E}\nu_1(B \cap G)$ for each Borel set $B \subset \mathbb{R}^2$ and for some (linear) intensities $\lambda_\ell, \lambda'_\ell > 0$. Each point of X_L is assumed to be connected to its closest neighbor in X_H, where two different meanings of 'closeness' are considered: either with respect to the Euclidean distance (case (e)), or in a graph–theoretic sense, i.e., along the edges of G (case (g)).

In applications, e.g. to hierarchical telecommunication networks, the edge set G can represent the underlying infrastructure, for instance, an inner–city street system. The Cox processes X_H and X_L can then describe the locations of (higher– and lower–level) network components. In this case, one is especially interested in the distribution of the typical shortest–path length C^* along the edge set between the points of X_L and their closest neighbors in X_H, which is an important performance characteristic in cost and risk analysis as well as in strategic planning of wired telecommunication.

Even for simple examples of connected and stationary geometric graphs G in $\mathbb{R}^2$, the distribution of C^* is not known analytically. However, asymptotic results can be derived if the graph G becomes unboundedly sparse and dense, respectively. In particular, we show that the limit distributions of C^* do not depend on the selected 'closeness' scenarios (e) or (g). Furthermore, for g-closeness, we show that the distribution of C^* does not depend on λ'_ℓ and decreases stochastically in λ_ℓ. It seems to be an open problem whether the latter property is also true for the case of e-closeness. On the other hand, it can be shown that under g-closeness, the distribution of C^* is stochastically smaller than under e-closeness.

Recently, in [4], the limit distributions of C^* have been determined for e-closeness, where it has been assumed that G is the edge set of a stationary Poisson–type tessellation with bounded convex cells. We extend these results performing an asymptotic analysis for typical shortest–path lengths on random geometric graphs induced by aggregate Poisson–Voronoi tessellations whose cells can be non-convex (see [2]). Furthermore, we consider β–skeletons on homogeneous Poisson point processes, which can have non–convex cells as well as dead ends (see [1]).

References:

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Gerd E. Schröder-Turk, University of Erlangen-Nürnberg

Minkowski tensors for morphology analysis of open-cell foams and porous materials

Joint work with Michael M. Klatt, Klaus Mecke, *et al.*

Minkowski tensors are the generalization of Minkowski functionals (or intrinsic volumes) to tensor-valued quantities. As their scalar counterparts, they can be usefully applied to the physically-relevant characterization of a shape of a given body. Due to their tensorial nature, they are hence particularly suited for the shape characterization w.r.t. orientation-dependent or anisotropic physical quantities, such as fluid permeabilities, effective mechanical properties, etc. Specifically for three-dimensional systems, it can be shown that there are six linearly-independent Minkowski tensors of rank two, denoted $W_\nu^{r,s}$ (ignoring another four given as products of scalar Minkowski functionals and the unit tensor); these tensors can be written as integrals of dyadic products of surface normal vectors and position vectors over a body K or its bounding surface ∂K. >From each of these tensors $W_\nu^{r,s}$ scalar measures of anisotropy can be derived e.g. as the minimal to maximal eigenvalue ratio, denoted $\beta_\nu^{r,s}$, each quantifying a different aspect of the body's anisotropy. This presentation will review the use of these shape indices for the physical study of several spatially structured materials, including granular, cellular and porous materials. In particular, we present an analysis of Boolean models of non-spherical particles with an orientation bias (i.e. particle positions are uncorrelated and uniformly distributed, but particle orientations are drawn from a non-uniform distribution of tunable width). This study reveals in particular that in this realistic model system for porous materials, the different aspects of anisotropy (quantified by the different Minkowski tensors) is qualitatively different.

Matthias Schulte, University of Osnabrück

Limit theorems for U-statistics of Poisson point processes

Joint work with Matthias Reitzner and Christoph Thäle

In Stochastic Geometry functionals of Poisson point processes often converge in distribution to a Gaussian random variable. In this talk, limit theorems for a broad class of these functionals, namely Poisson U-statistics, are presented.
We call a sum over all k-tuples of a Poisson point process a Poisson U-statistic. An Example for a Poisson U-statistic in Stochastic Geometry is the volume of the intersection process of Poisson hyperplanes in a convex body. The Wiener-Itô chaos expansion of such a Poisson U-statistic is a finite sum of multiple Wiener-Itô integrals. This representation enables us to compute exactly the variance and higher moments. Combining the expansion and a recent result for the normal approximation of Poisson functionals due to Peccati, Solé, Taqqu and Utzet yields central limit theorems including a rate of convergence.

Vadim Shcherbakov, Moscow State University

Statistical inference for cooperative sequential adsorption and related problems

Cooperative sequential adsorption (CSA) models are widely used for modelling adsorption processes in physics and chemistry. Mathematically a CSA model is a probabilistic model for sequential packing and deposition and a typical CSA model is formulated as a random finite sequential allocation of particles (points, or, in general, objects of arbitrary shape) in a bounded region of space (the observation window). CSA dynamics seem to be relevant to many applications such as modelling the irreversible spread of epidemics or biological growth. Motivated by these applications we developed in [1] and [2] statistical inference for a parametric class of CSA. Our approach is based on maximum likelihood estimation. In turn, asymptotic analysis of maximum likelihood estimators is based on the observation that the model likelihood is determined by statistics of a special type (sums of locally determined functionals over a configuration of points). This allows us to analyse the asymptotics of the estimators by exploiting recent developments in the modern probability, namely, the limit theory for random sequential packing and deposition. Our method can be applied in the same way for any spatial-temporal model which has a likelihood structure similar to the one of CSA. Also, the method appears to be applicable for establishing asymptotic properties of pseudo-likelihood estimators for finite point processes.

References:

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Malte Spiess, Ulm University

Asymptotic properties of the approximate inverse estimator for directional distributions

Joint work with Martin Riplinger

We consider the estimator for the directional distribution of stationary fiber processes which

was introduced in the recent article [1]. The estimation approach is based on the well-known stereological idea of counting the intersection points of the process with test hyperplanes in a bounded observation window. To derive the directional distribution from this information, it is necessary to invert the cosine transform. Since in practice only finitely many test hyperplanes can be considered, we use the method of the approximate inverse to ensure the numerical stability of the operation.

In this talk, we consider Poisson line processes and analyze the asymptotic properties of the estimator, including pointwise weak convergence and weak convergence of the L^2-distance. Finally, we derive Berry-Esseen bounds and large deviation results for the pointwise convergence.

References:

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Evgeny Spodarev, Ulm University

Extrapolation of symmetric stable random fields

Joint work with Wolfgang Karcher and Elena Shmileva

The theory of kriging developed since 1952 for the extrapolation of wide sense stationary random fields is based on minimizing the mean square deviation between the true (unobserved) value of the field and its extrapolator.

In many applications, stationary random fields without a finite second moment play a crucial role. The matter of this talk is to give some new methods for the extrapolation of symmetric α-stable random fields ($\alpha \in (1, 2)$) and compare them with the existing ones. Most of these methods yield linear estimators that are optimal in some specified sense. We prove the existence and uniqueness of these estimators for symmetric stable moving averages. In the proofs, ideas of the representation of symmetric stable laws by zonoids (first given by I. Molchanov (2009)) are used.

Christoph Thäle, University of Osnabrück

Iteration stable tessellations: Variances and limit theory

Joint work with Tomasz Schreiber

Iteration stable random tessellations (STIT tessellations for short) form a relatively new class of random tessellations of the Euclidean space. They have attracted considerable interest in stochastic geometry, because many of its parameters allow an explicit calculation. STIT tessellations clearly show the potential to become a new mathematical reference model beside the classical Poisson hyperplane or Poisson-Voronoi tessellation. In my talk I show how martingale theory helps to develop a deep understanding of the second-order structure as well as of the asymptotic regime of these tessellations. In particular, integral-geometric tools may be used to provide explicit expressions for certain variances and covariances, both exact and asymptotic. This together with the general martingale limit theory puts us into the position to establish central limit theorems for homogeneous functionals of random STIT tessellations. In contrast to

many other models considered in stochastic geometry, our theory leads to Gaussian limits in the planar case, whereas non-Gaussian limits show up in all higher dimensions.

Ólöf Thórisdóttir, Aarhus University

Variance of surface area estimators from central sections

Quite recently, Cruz-Orive [1] defined a new surface area estimator, the pivotal estimator, which only requires measurements in isotropic central sections through a reference point. Hence the surface area of a three-dimensional object can be unbiasedly estimated using measurements in two dimensions avoiding three dimensional scanning of the object. The estimator is based on the invariator principle [1], which shows how a line in an isotropic two-dimensional plane can be generated such that it is a motion invariant line in three dimensions. The variance of the pivotal estimator has only been studied to a very limited extent and the talk will extend this analysis. We will show how the variance of the pivotal estimator can be decomposed into different contributions when the object of interest is a convex body (i.e. a compact, convex set). In the case of balls explicit analytic formulas for these different variance contributions are obtained whereas numerical results are given in the case of ellipsoids. Based on the results for ellipsoids we will discuss some variance reduction ideas, continuing the work of Cruz-Orive [2]. Some of them require a closer examination of the sample, others require a modification of the sampling design. Ideas that will be suggested include variance reduction by measuring the support function in the isotropic plane and variance reduction by systematic sampling in the isotropic plane. We will in particular discuss different versions of systematic sampling in the plane. Theoretical results and simulated studies are used to illustrate the variance reduction obtained by applying the new, improved estimators.

References:

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Steffen Winter, Karlsruhe Institute of Technology

Volume and surface area of parallel sets

Joint work with Jan Rataj

Fractal sets, like (random) self-similar sets or the Brownian path in R^d, cannot be analysed by standard geometric means. One possibility to overcome this is to approximate the sets by their r-parallel sets and consider the asymptotic behavior of certain geometric characteristics as r tends to 0.

The observation that the r-parallel sets of bounded sets have a rectifiable boundary, allows to improve some known results on volume and boundary surface area of the r-parallel sets and, in particular, on their asymptotic behaviour as $r \to 0$. We show that there is a close relation between the Minkowski content and the corresponding rescaled limit of the boundary surface area, called the S-content. These two geometric quantities appear naturally as special cases in the framework of fractal curvatures.

Some applications to random closed sets, in particular the Wiener sausage, and to self-similar

fractal sets are discussed.

Rasmus Waagepetersen, Aalborg University

Generalized estimating equations for spatial point processes

A spatial point process X on $S \subseteq \mathbb{R}^d$ may be viewed equivalently as a locally finite random set $X \subset S$ or as a stochastic process of random count variables $N(B)$ indexed by bounded sets $B \subseteq S$. The latter point of view is useful for constructing estimating functions by adapting existing methodology for count variables. Such estimating functions provide computationally easy alternatives to maximum likelihood estimation which can be computationally very challenging e.g. for Cox and cluster point processes. In this talk we will discuss how the idea of generalized estimating equations (GEE) can be adapted to the setting of spatial point processes. We consider in particular the limiting form of the GEE when the sizes of the index sets B tend to zero, computational issues and asymptotic properties of estimates.

Joseph Yukich, Lehigh University, Bethlehem, Pennsylvania

Non-parametric estimation of surface integrals

The talk extends joint work with R. Jiménez (Annals of Statistics, 2011, 39, 232-260)

Let $X_i, i \geq 1$, be an i.i.d. uniform sample in $[0,1]^d, d \geq 2$, and let $G \subset [0,1]^d$ have boundary ∂G, which in general is unknown and not smooth. We review ways to use the sample X_i to (i) reconstruct ∂G and (ii) estimate the volume content of ∂G with consistent estimators. More generally, if $h : [0,1]^d \to \mathbb{R}$ is unknown, but the values $h(X_i), i \geq 1$, are knowable, then we consider estimators of the surface integral $\int_{\partial G} h dx$, and we establish their consistency and asymptotic normality.

Johanna Ziegel, Heidelberg University

Precision estimation for stereological volumes

Volume estimators based on Cavalieri's principle are widely used in the biosciences. For example in neuroscience, where volumetric measurements of brain structures are of interest, systematic samples of serial sections are obtained by magnetic resonance imaging or by a physical cutting procedure. The volume v is then estimated by $\hat{v}$, which is the sum over the areas of the structure of interest in the section planes multiplied by the width of the sections, $t > 0$.

Assessing the precision of such volume estimates is a question of great practical importance, but statistically a challenging task due to the strong spatial dependence of the data and typically small sample sizes. In this talk an overview of classical and new approaches to this problem will be presented. A special focus will be given to some recent advances on distribution estimators and confidence intervals for $\hat{v}$; see Hall and Ziegel (2011).

References:

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Markéta Zikmundová, Charles University in Prague

Spatio-temporal model for a random set given by a union of interacting discs

Joint work with Kateřina Staňková Helisová and Viktor Beneš

We define a spatio-temporal random set model based on the union of interaction discs. A state space model is used for the temporal change of parameters corresponding to integral–geometric variables of the random set. The evolution of parameters of the state space model is estimated using sequential Monte Carlo. Our aim is to compare the particle filter and MCMC maximum likelihood estimation of the parameters.

References:

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