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Limit theory for non-linear statistics of random geometric data

Joint with B. Błaszczyszyn and D. Yogeshwaran

Statistics of random geometric data $\mathcal{X} \subset \mathbb{R}^d$ often consist of sums of spatially dependent terms admitting the representation

$$\sum_{x \in \mathcal{X}} \xi(x, \mathcal{X}), \quad (1)$$

where the \mathbb{R} -valued *score function* ξ , defined on pairs (x, \mathcal{X}) , $x \in \mathcal{X}$, represents the *interaction* of x with respect to the data \mathcal{X} . The sums (1) typically describe a global feature of the geometric data in terms of a sum of local contributions $\xi(x, \mathcal{X})$, $x \in \mathcal{X}$. Functionals and non-linear statistics arising in random graphs and data analysis may often be cast in the form (1) for appropriately chosen ξ . This includes statistics of simplicial complexes, Morse critical points, the Euler-Poincaré characteristic, as well as total edge length and clique counts in random graphs. If the data \mathcal{X} is the realization of a clustering point process and if ξ is ‘locally’ determined (i.e., stabilizing), then we establish general expectation and variance asymptotics as well as central limit theorems for the suitably scaled sums

$$\sum_{x \in \mathcal{X} \cap W_n} \xi(x, \mathcal{X} \cap W_n) \quad (2)$$

as $W_n \uparrow \mathbb{R}^d$. Given a score function ξ which is locally determined we thus obtain general limit theorems for the sums (2) whenever \mathcal{X} is given by a determinantal or permanental point process with a fast decreasing kernel (e.g. the Ginibre ensemble), the zero set of a Gaussian entire function, or rarified Gibbsian input. This extends the existing literature treating the limit theory of sums of stabilizing scores of Poisson and binomial input. In the setting of clustering point processes, it also extends work of Nazarov and Sodin as well as work of Soshnikov, which are confined to linear statistics $\sum_{x \in \mathcal{X} \cap W_n} \xi(x)$. The talk is based on joint work with B. Błaszczyszyn and D. Yogeshwaran.